MATH 152

Final Exam

Fall 1997

Version B

Student (Print) _____
Last, First Middle

Student (Sign) _____

Student ID _____

Instructor ____P. Yasskin_____

Section _____

1-13	
14	
15	
16	
17	
TOTAL	

Part I is multiple choice. There is no partial credit. You may not use a calculator.

Part II is work out. Show all your work. Partial credit will be given. You may use your calculator.

Part I: Multiple Choice (5 points each)

There is no partial credit. You may not use a calculator. You have 1 hour.

1. Find the area between the parabola $y = x^2 - 2x$ and the line $y = x$.

a. $-\frac{10}{3}$

b. $\frac{7}{6}$

c. $\frac{13}{6}$

d. $\frac{10}{3}$

e. $\frac{9}{2}$

2. Find the plane tangent to the hyperbolic paraboloid $z = x^2 - y^2$ at the point $(2,1,3)$. Its z-intercept is $z =$

a. 1

b. 0

c. -1

d. -2

e. -3

3. In the partial fraction expansion $\frac{x^2+4}{x(x-2)^2} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$ the coefficients are

a. $A = -1, B = 1, C = 2$

b. $A = -1, B = 0, C = 2$

c. $A = 1, B = 0, C = 4$

d. $A = 0, B = 1, C = -4$

e. $A = 1, B = -1, C = -4$

4. The area in the first quadrant between the hyperbola $y = \frac{3}{x}$ and the line $y = 4 - x$ is rotated about the y-axis. Find the volume of the solid swept out.

a. $\pi \int_1^3 \left(4 - x - \frac{3}{x}\right)^2 dx$

b. $2\pi \int_1^3 \left(4 - x - \frac{3}{x}\right)^2 dx$

c. $2\pi \int_1^3 \left(4 - x - \frac{3}{x}\right) dx$

d. $2\pi \int_1^3 (4x - x^2 - 3) dx$

e. $2\pi \int_1^3 (4 - x)^2 - \left(\frac{3}{x}\right)^2 dx$

5. Which limit does not exist?

a. $\lim_{(x,y)\to(0,0)} (2x^2 + y^2)$

b. $\lim_{(x,y)\to(0,0)} (2x^2 - y^2)$

c. $\lim_{(x,y)\to(0,0)} \frac{2x^2 - y^2}{x^2 + y^2}$

d. $\lim_{(x,y)\to(0,0)} \frac{2x^4 + x^2y^2}{x^2 + y^2}$

e. $\lim_{(x,y)\to(0,0)} \frac{2x^4 - x^2y^2}{x^2 + y^2}$

6. $\int_1^{\sqrt{2}} x^3 \sqrt{x^2 - 1}\, dx =$

a. $\frac{2}{15}$

b. $\frac{8}{15}$

c. $\frac{22}{15}\sqrt{2} - \frac{8}{15}$

d. $\frac{8}{15}(2^{5/4} + 2^{3/4})$

e. $\frac{8}{15}(2^{5/4} + 2^{3/4} - 1)$

7. An airplane is circling above an airport, clockwise as seen from above. Thus its wings are banked with the right wing lower than the left wing. In what direction does the binormal $\vec{B}$ point?

a. horizontally toward the center of the circle

b. vertically up

c. vertically down

d. along the left wing

e. along the right wing

8. Solve the differential equation $\frac{dy}{dx} = 28x\sqrt{y}$. If $y(0) = 4$, then $y(1) =$

a. 81
b. 53
c. 49
d. 11
e. 9

9. A $50\,\text{ft}$ cable is hanging from the roof down the side of a tall building. If the cable weighs $3\,\text{lb/ft}$, how much work is done to lift the cable to the roof?

a. $7500\,\text{ft-lb}$
b. $3750\,\text{ft-lb}$
c. $1875\,\text{ft-lb}$
d. $150\,\text{ft-lb}$
e. $75\,\text{ft-lb}$

10. A nuclear power plant is producing a radioactive isotope X at the rate of $75\,\text{kg/yr}$. Let $N(t)\,\text{kg}$ be the amount of X at the power plant at time t. A known fact is that X decays at the rate of $50N(t)\,\text{kg/yr}$. Write the differential equation to be solved for $N(t)$.

a. $\frac{dN}{dt} = 50N - 75$
b. $\frac{dN}{dt} = 75 - 50N$
c. $\frac{dN}{dt} = 50N - 75t$
d. $\frac{dN}{dt} = 75t - 50N$
e. $\frac{dN}{dt} = -25N$

11. The function $f(x,y)$ has the values given below. Estimate $\frac{\partial f}{\partial y}(2.1, 3.0)$.

$f(2.0, 3.2) = 3$ $\quad f(2.1, 3.2) = 7$ $\quad f(2.2, 3.2) = 9$

$f(2.0, 3.1) = 2$ $\quad f(2.1, 3.1) = 4$ $\quad f(2.2, 3.1) = 8$

$f(2.0, 3.0) = 1$ $\quad f(2.1, 3.0) = 2$ $\quad f(2.2, 3.0) = 5$

a. 2
b. 10
c. 15
d. 20
e. 30

12. Which of the planes described below contain the lines:

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-1}{4} \quad \text{and} \quad \frac{x-1}{4} = \frac{y-2}{3} = \frac{z-1}{2} \ ?$$

a. $x - 2y + z = -2$
b. $x - 2y + z = 2$
c. $2x - y + z = -1$
d. $2x - y + z = -1$
e. $x - y + z = 0$

13. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. If the radius is measured as $r = 3 \pm .2$, and the height is measured as $h = 4 \pm .1$, then the volume is computed as $V = 12\pi \pm \Delta V$ where the error in the measurement of the volume is $\Delta V =$

a. 1.1π
b. 1.7π
c. 1.9π
d. 2.2π
e. 2.7π

Part II: Work Out

Show all your work. Partial credit will be given.
You may use your calculator but only after 1 hour.

14. (10 points) Consider the curve $\boldsymbol{r}(t) = (1 - t^2, 1 + t^2, t)$. Compute each of the following:

a. velocity

$\boldsymbol{v} =$ _____

b. speed (Simplify.)

$|\boldsymbol{v}| =$ _____

c. acceleration

$\boldsymbol{a} =$ _____

d. curvature

$\kappa =$ _____

e. tangential acceleration

$a_T =$ _____

f. normal acceleration

$a_N =$ _____

15. (10 points) Compute $\int_0^{3/4} \frac{1}{\sqrt{1+x^2}}\,dx$

16. (10 points) A comet has just passed by the sun.
Its current position is $(x,y) = (3 \times 10^7 \text{ mi}, 4 \times 10^7 \text{ mi})$
and its current velocity is $\boldsymbol{v} = \left(\frac{dx}{dt}, \frac{dy}{dt} \right) = \left(4 \times 10^7 \frac{\text{mi}}{\text{day}}, 7 \times 10^7 \frac{\text{mi}}{\text{day}} \right)$.
Hence the current distance from the sun is $R = \sqrt{x^2 + y^2} = 5 \times 10^7 \text{ mi}$.
Find the rate at which the distance from the sun is increasing.

17. (5 points) This is the direction field of a certain differential equation. Draw in the solution $y = y(x)$ which satisfies the initial condition $y(2) = 3$.

1.