#### MATH 152, SPRING 2006 COMMON EXAM II - VERSION A

LAST NAME, First Name (print):

INSTRUCTOR:

SECTION NUMBER: \_\_\_\_\_

UIN: \_\_\_\_\_

### **DIRECTIONS:**

- 1. The use of a calculator, laptop or computer is prohibited.
- 2. In Part 1 (Problems 1-10), mark the correct choice on your ScanTron form No. 815-E using a No. 2 pencil. For your own records, also record your choices on your exam! ScanTrons will be collected from all examinees after 90 minutes and will not be returned.
- 3. In Part 2 (Problems 11-14), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
- 4. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

# THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: \_\_\_\_\_

### DO NOT WRITE BELOW!

Question	Points Awarded	Points	
1-10		50	
11		12	
12		12	
13		14	
14		12	
		100	

### PART I

1. (5 pts) The curve C is given by the parametric equations  $x = t^2$ ,  $y = t^2 + t$  for  $0 \le t \le 1$ . Which integral gives the length of the curve C?

(a) 
$$\int_{0}^{1} \sqrt{t^{2} + 2t^{3} + 2t^{4}} dt$$
  
(b)  $\int_{0}^{1} \sqrt{1 + t^{2} + 2t^{3} + 2t^{4}} dt$   
(c)  $\int_{0}^{1} \sqrt{1 + 4t + 8t^{2}} dt$   
(d)  $\int_{0}^{1} \sqrt{2 + 4t + 8t^{2}} dt$   
(e)  $\int_{0}^{1} \sqrt{2 + 4t} dt$ 

2. (5 pts) Which differential equation is <u>not</u> separable?

(a) 
$$\frac{dy}{dx} = (\sin x)(\cos y)$$
  
(b) 
$$\frac{dy}{dx} = xy + x^2 y$$
  
(c) 
$$\frac{dy}{dx} = xy^2 + x^2 y$$
  
(d) 
$$\frac{dy}{dx} = e^{x+y}$$
  
(e) 
$$\frac{dy}{dx} = 1 + x + y + xy$$

- 3. (5 pts) An integrating factor for the differential equation  $y' + (2\sin 2x)y = \cos 4x$  is
  - (a)  $-\cos(2x)$
  - (b)  $\sin(2x)$
  - (c)  $e^{\frac{1}{4}\sin(4x)}$
  - (d)  $e^{-\cos(2x)}$
  - (e)  $e^{\sin(2x)}$

- 4. (5 pts) A tank contains 100 L of pure water. Brine that contains 0.1 kg of salt per liter enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. If y(t) is the quantity of salt in kilograms dissolved in the tank at time t, then y satisfies
  - (a)  $\frac{dy}{dt} = 1 \frac{y}{10}$ , y(0) = 0.1(b)  $\frac{dy}{dt} = 1 - \frac{y}{10}$ , y(0) = 0(c)  $\frac{dy}{dt} = -\frac{y}{10}$ , y(0) = 0
  - (d)  $\frac{dy}{dt} = -\frac{y}{100}$ , y(0) = 0.1(e)  $\frac{dy}{dt} = 10 - \frac{y}{100}$ , y(0) = 0

5. (5 pts) Which statement about  $\int_1^\infty \frac{x}{1+x^4} dx$  is true?

(a) converges by comparison to  $\int_{1}^{\infty} \frac{1}{x^3} dx$ 

(b) diverges by comparison to 
$$\int_{1}^{\infty} \frac{1}{x^3} dx$$

(c) converges by comparison to 
$$\int_{1}^{\infty} \frac{1}{x} dx$$

(d) diverges by comparison to 
$$\int_{1}^{1} \frac{1}{x} dx$$

(e) diverges by comparison to 
$$\int_1 x \, dx$$

6. (5 pts) Find the x-coordinate of the center of mass of the semi-circular plate of radius 4, shown as the region  $\mathbf{R}$  in the accompanying figure.



(a) 36 (b)  $\frac{16}{3\pi}$ (c) 54 (d)  $\frac{27}{\pi}$ 

(e)  $\pi$ 

For problems 7 and 8 refer to the following table of function values.

x	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
f(x)	1.00	0.50	0.25	0.75	0.40	0.30	0.20	0.10	0.70

- 7. (5 pts) Calculate the trapezoid approximation of  $\int_0^2 f(x) dx$  for the partition of  $0 \le x \le 2$  into four equal sub-intervals.
  - (a) 1.60
  - (b) 1.75
  - (c) 1.00
  - (d) 0.825
  - (e) 0.85

- 8. (5 pts) Calculate the midpoint approximation of  $\int_0^2 f(x) dx$  for the partition of  $0 \le x \le 2$  into four equal sub-intervals.
  - (a) 1.00
  - (b) 0.85
  - (c) 0.825
  - (d) 1.60
  - (e) 1.75

9. (5 pts) Find the general solution of  $\frac{dy}{dx} = (2+3x^2)(y^2+1)$ .

- (a)  $y = 2x + x^3 + C$
- (b)  $y = \sin(2x + x^3 + C)$
- (c)  $y = \sin(2x + x^3)$
- (d)  $y = \tan(2x + x^3) + C$
- (e)  $y = \tan(2x + x^3 + C)$

10. (5 pts) If  $\frac{dy}{dx} = 2y$  and y(0) = 4, then y(1) =(a)  $2e^4$ (b)  $4e^4$ (c)  $4 \ln 2$ (d)  $4e^2$ (e)  $2e^2$ 

## PART II

11. (12 pts) Calculate the length of the curve 
$$y = \frac{x^2}{4} - \frac{\ln x}{2}$$
 from  $x = 1$  to  $x = 2$ .

12. (12 pts) Solve the differential equation  $xy' + 2y = x^3$  with the initial condition y(-1) = 2.

13. Evaluate each of the following improper integrals if it converges or explain why it diverges.

(a) (7 pts) 
$$\int_0^\infty \frac{x}{1+x^2} \, dx$$

(b) (7 pts)  $\int_0^2 \frac{1}{\sqrt{4-x^2}} dx$ 

14. (12 pts) Find the area of the surface obtained by rotating the curve  $y = x^3$  over  $0 \le x \le 1$  about the x-axis.

End of exam