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Instructor

## MATH 152, Fall 2007 Common Exam 3 Test Form A

Instructions:
You may not use notes, books, calculator or computer.
Part I is multiple choice. There is no partial credit.
Mark the Scantron with a \#2 pencil. For your own records, also circle your choices in this exam. Scantrons will be collected after 90 minutes and may not be returned.

Part II is work out. Show all your work. Partial credit will be given.
THE AGGIE CODE OF HONOR:
An Aggie does not lie, cheat or steal, or tolerate those who do.

| For Dept use Only: |  |
| :---: | ---: |
| $1-11$ | $/ 55$ |
| 12 | $/ 15$ |
| 13 | $/ 12$ |
| 14 | $/ 12$ |
| 15 | $/ 12$ |
| TOTAL |  |

## Part I: Multiple Choice (5 points each)

There is no partial credit.

1. Compute $\lim _{n \rightarrow \infty} \frac{3^{n}}{1+4^{n}}$
a. 0
b. $\frac{3}{4}$
c. $\frac{1}{1-\frac{3}{4}}$
d. $\frac{\frac{3}{5}}{1-\frac{3}{4}}$
e. $\infty$
2. Compute $\sum_{n=1}^{\infty} \frac{2^{n+1}}{5^{n}}$
a. 4
b. 10
c. $\frac{4}{3}$
d. $\frac{10}{3}$
e. The series diverges.
3. Compute $S=\sum_{n=1}^{\infty}\left(3^{1 / n}-3^{1 /(n+1)}\right)$
a. 3
b. 2
c. 1
d. 0
e. The series diverges.
4. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n}}{\sqrt{n}}$ is
a. absolutely convergent.
b. absolutely convergent but not convergent.
c. convergent but not absolutely convergent.
d. divergent.
5. Find the coefficient of $x^{6}$ in the Maclaurin series for $f(x)=\sin \left(x^{2}\right)$.
a. 0
b. $\frac{1}{24}$
c. $-\frac{1}{6}$
d. $-\frac{1}{2}$
e. 1
6. Compute $\sum_{n=0}^{\infty} \frac{(-2)^{n}}{n!}$ HINT: Think about a known Maclaurin series.
a. $\frac{1}{e^{2}}$
b. $\frac{-1}{e^{2}}$
c. $\frac{1}{2}$
d. $-e^{2}$
e. The series diverges.
7. Compute $\lim _{x \rightarrow 0} \frac{\cos \left(x^{3}\right)-1+\frac{1}{2} x^{6}}{x^{12}}$
a. $-\frac{1}{24}$
b. $-\frac{1}{4}$
c. $\frac{1}{4}$
d. $\frac{1}{6}$
e. $\frac{1}{24}$
8. The series $S=\sum_{n=0}^{\infty} \frac{3^{n}}{1+4^{n}} \quad$ satisfies
a. $S=0$
b. $0<S<4$
c. $S=4$
d. $S>4$
e. The series diverges.
9. Find the equation of the sphere centered at $(1,2,3)$ which passes through the point ( $-3,0,-1$ ).
a. $(x-3)^{2}+(y)^{2}+(z+1)^{2}=6$
b. $(x+3)^{2}+(y)^{2}+(z-1)^{2}=36$
c. $(x+3)^{2}+(y)^{2}+(z-1)^{2}=6$
d. $(x-1)^{2}+(y-2)^{2}+(z-3)^{2}=36$
e. $(x-1)^{2}+(y-2)^{2}+(z-3)^{2}=6$
10. A vector $\vec{u}$ has length 3. A vector $\vec{v}$ has length 4 . The angle between them is $45^{\circ}$. Find $\vec{u} \cdot \vec{v}$.
a. 3
b. 12
c. $12 \sqrt{2}$
d. 6
e. $6 \sqrt{2}$
11. A triangle has vertices $A=(1,2,3), B=(2,3,3)$ and $C=(1,3,2)$. Find the angle at $A$.
a. $0^{\circ}$
b. $30^{\circ}$
c. $45^{\circ}$
d. $60^{\circ}$
e. $90^{\circ}$

## Part II: Work Out (points indicated)

Show all your work. Partial credit will be given.
12. (15 points) For each series, determine if it is convergent or divergent. Be sure to identify the Convergence Test and check out its hypotheses.
a. $\sum_{n=2}^{\infty} n e^{\left(-n^{2}\right)}$
b. $\sum_{n=2}^{\infty} \frac{n^{5}-3}{n^{6}+1}$
c. $\sum_{n=2}^{\infty}(-1)^{n} \frac{n}{n-1}$
13. (12 points) Find the radius and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n}}{n 3^{n}}(x-4)^{n}$.
14. (12 points)
a. Find the $2^{\text {nd }}$ degree Taylor polynomial, $T_{2}(x)$, for $f(x)=\frac{1}{\sqrt{x}}$ at $x=4$.
b. Use this polynomial to approximate $\frac{1}{\sqrt{6}}$. Do not simplify.
c. Estimate the error in your approximation in part (b). Do not simplify. Justify your answer.
15. (12 points)
a. Find a Maclaurin series for $e^{-x^{2}}$.
b. Write the integal $\int_{0}^{0.1} e^{-x^{2}} d x$ as an infinite series using summation notation.
c. Use the series from part (b) to approximate the integal $\int_{0}^{0.1} e^{-x^{2}} d x$ to within $\pm 10^{-5}$. Explain how you know your estimate is correct to within $\pm 10^{-5}$.

