MATH 152, FALL SEMESTER 2008
COMMON EXAMINATION I - VERSION A

Name (print):____

Signature: ____

Instructor's name: ____

Section No: ____

INSTRUCTIONS

1. In Part 1 (Problems 1–10), mark your responses on your ScanTron form using a No: 2 pencil. *For your own record, mark your choices on the exam as well.*

2. Calculators **should not be used** throughout the examination.

3. In Part 2 (Problems 11–14), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form**.

QN	PTS
1–10	
11	
12	
13	
14	
TOTAL	

Part 1 – Multiple Choice (50 points)

Read each question carefully; each problem is worth **5 points**. *Mark your responses on the ScanTron form and on the exam itself.*

1. Evaluate $\int_1^e \frac{\sqrt{\ln x}}{x}\, dx$.

(a) 3/2

(b) 2/3

(c) 1/2

(d) $-\infty$

(e) $\frac{2(e^{3/2}-1)}{3}$

2. Evaluate $\int_0^1 x(2x-1)^7\, dx$.

(a) 1/9

(b) 2/9

(c) 1/18

(d) −1/72

(e) 17/72

3. Compute the area under the curve $y = xe^{-x}$, between $x = 0$ and $x = 1$.

(a) $1 - 2e^{-1}$

(b) $1 + 2e^{-1}$

(c) $\frac{1-e^{-1}}{2}$

(d) $\frac{e^{-1}-1}{2}$

(e) $2e^{-1} - 1$

4. Compute the area enclosed by the parabola $y = x^2$ and the straight line $y = 2x$.

(a) 2/3

(b) 3/4

(c) 1/2

(d) 4/3

(e) 1

5. A force of 10 lb is required to hold a spring stretched 1/3 ft beyond its natural length. How much work is done in stretching it from its natural length to 1/2 ft beyond its natural length? (Recall and use *Hooke's Law*: the force required to maintain a spring attached x units beyond its natural length is given by $f(x) = kx$, where k is a constant.)

(a) 15/2 ft-lb

(b) 15 ft-lb

(c) 15/4 ft-lb

(d) 45/2 ft-lb

(e) 45 ft-lb

6. Use the trigonometric identity $2\sin A\cos B \equiv \sin(A+B)+\sin(A-B)$ to compute the indefinite integral $\int \sin 3t \cos 2t\, dt$.

(a) $-\dfrac{\cos 5t}{5} - \cos t + C$

(b) $\dfrac{\cos 5t}{5} + \sin t + C$

(c) $\dfrac{\cos 5t}{10} + \dfrac{\sin t}{2} + C$

(d) $-\dfrac{\sin 5t}{10} - \dfrac{\sin t}{2} + C$

(e) $-\dfrac{\cos 5t}{10} - \dfrac{\cos t}{2} + C$

7. Compute $\int \tan^3\theta \sec\theta \, d\theta$.

(a) $\dfrac{\sec^3\theta}{3} - \sec\theta + C$

(b) $\dfrac{\tan^3\theta}{3} + C$

(c) $\dfrac{\tan^3\theta}{3} - \tan\theta + C$

(d) $\left(\dfrac{\tan^4\theta}{4}\right) \ln|\sec\theta + \tan\theta| + C$

(e) $\tan^4\theta \sec\theta + 3\tan^2\theta \sec^3\theta + C$

8. Compute $\displaystyle\int \frac{dx}{(3-x)(3+x)}$.

(a) $\ln|9 - x^2| + C$

(b) $-\dfrac{1}{3} \ln\left|\dfrac{x-3}{x+3}\right| + C$

(c) $-\dfrac{1}{6} \ln\left|\dfrac{x-3}{x+3}\right| + C$

(d) $\ln\left|\dfrac{x-3}{x+3}\right| + C$

(e) $(\ln|3-x|)(\ln|3+x|) + C$

9. Which of the following is the correct partial-fraction decomposition for the rational function $\dfrac{x^4+x+1}{(x^2-1)^2(x^2+1)^2}$?

(a) $\dfrac{A}{x^2-1} + \dfrac{B}{x^2-1} + \dfrac{C}{x^2+1} + \dfrac{D}{x^2+1}$

(b) $\dfrac{A}{x^2-1} + \dfrac{B}{(x^2-1)^2} + \dfrac{C}{x^2+1} + \dfrac{D}{(x^2+1)^2}$

(c) $\dfrac{A}{(x-1)^2} + \dfrac{B}{(x+1)^2} + \dfrac{Cx+D}{x^2+1} + \dfrac{Ex+F}{(x^2+1)^2}$

(d) $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x+1} + \dfrac{D}{(x+1)^2} + \dfrac{E}{x^2+1} + \dfrac{F}{(x^2+1)^2}$

(e) $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x+1} + \dfrac{D}{(x+1)^2} + \dfrac{Ex+F}{x^2+1} + \dfrac{Gx+H}{(x^2+1)^2}$

10. Suppose that f is a continuous function defined on $[0, \infty)$, and that the average value of f over the interval $[0, t]$ equals $t^2 + 1$ for every $t > 0$. Find f.

(a) $f(x) = x^2 + 1$

(b) $f(x) = 2x$

(c) $f(x) = \dfrac{x^3}{3} + x$

(d) $f(x) = 3x^2 + 1$

(e) $f(x) = x^3 + x$

Part 2 (55 points)

Present your solutions to the following problems (11–14) in the space provided. **Show all your work** *neatly and concisely, and* **indicate your final answer clearly**. *You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.*

11. (`7 points`) Let $\mathcal{S}$ be the solid whose base is the triangle with vertices $(0,0)$, $(1,0)$, and $(0,2)$, and whose cross sections perpendicular to the y-axis are semicircles. Compute the volume of $\mathcal{S}$.

12. (`7 points`) Let $a > 0$, and let $\mathcal{R}$ be the region enclosed by $y = 2x^3$, the x-axis, and the line $x = a$. Let $\mathcal{S}$ denote the solid obtained by rotating $\mathcal{R}$ about the x-axis. Ascertain the value of a for which the volume of $\mathcal{S}$ is π times the area of $\mathcal{R}$.

13. (i) (`7 points`) Let $\mathcal{R}_t$ denote the region enclosed by the y-axis, the line $y = 1$, and the curve $y = \sqrt{x}$. Use the method of discs to compute the volume of the solid obtained by rotating the region $\mathcal{R}_t$ about the y-axis.

(ii) (`7 points`) Let $\mathcal{R}_b$ denote the region enclosed by the x-axis, the line $x = 1$, and the curve $y = \sqrt{x}$. Use the method of cylindrical shells to calculate the volume of the solid obtained by rotating the region $\mathcal{R}_b$ about the line $x = 1$.

14. Compute each of the following integrals:

(i) (9 points) $\int 4x \tan^{-1}(2x)\, dx$

(ii) (`9 points`) $\int x\sqrt{1-9x^2}\,dx$

(iii) (9 points) $\int \frac{(x-1)^2}{x^3+x}\,dx$