Name______________________________ Section____________

MATH 152 Honors FINAL EXAM Spring 2014

Sections 201-202 P. Yasskin

Multiple Choice: (14 problems, 4 points each)

1-14	/56
15	/22
16	/20
17	/4
18	/4
Total	/106

1. Find the area under the curve $y = \dfrac{1}{x^2+1}$ above the interval $[0,1]$.

a. $\dfrac{1}{4}$

b. $\dfrac{\pi}{4}$

c. $\dfrac{1}{2}$

d. 1

e. $\dfrac{\pi}{2}$

2. The region under the curve $y = \dfrac{1}{x^2+1}$ above the interval $[0,1]$ is revolved about the y-axis. Find the volume of the resulting solid.

a. $\pi \ln(2)$

b. $\pi \ln(2) - \pi$

c. $2\pi \ln(2)$

d. $4\pi \ln(2)$

e. $4\pi \ln(2) - 4\pi$

3. A plate with constant density ρ has the shape of the region below $y = \dfrac{1}{x^2+1}$ above the interval $[0,1]$. Find the x-coordinate of its center of mass.

a. $\dfrac{\pi}{2}\ln 2$

b. $\dfrac{2}{\pi}\ln 2$

c. $\dfrac{\pi}{2\ln 2}$

d. $\dfrac{2}{\rho\ln 2}$

e. $\dfrac{\rho}{2}\ln 2$

4. Compute $\displaystyle\int_0^{\pi/4} \frac{\sec^2\theta}{\tan^2\theta}\,d\theta.$

a. $-\infty$

b. -1

c. $\dfrac{1}{2}$

d. 1

e. ∞

5. Compute $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\,dx.$ HINT: $\sin(2\theta) = 2\sin\theta\cos\theta$

a. $\dfrac{1}{2}\arcsin x - \dfrac{x}{3}(1-x^2)^{3/2} + C$

b. $\dfrac{1}{2}\arcsin x + x\sqrt{1-x^2} + C$

c. $\dfrac{1}{2}\arcsin x - x\sqrt{1-x^2} + C$

d. $\dfrac{1}{2}\arcsin x - \dfrac{x}{2}\sqrt{1-x^2} + C$

e. $\dfrac{1}{2}\arcsin x + \dfrac{x}{2}\sqrt{1-x^2} + C$

6. Compute $\int (\ln x)^2 dx$. HINT: $u = (\ln x)^2$.

a. $x\ln^2 x - x^2 \ln x - \frac{1}{2}x^2 + C$

b. $x\ln^2 x - x^2 \ln x + \frac{1}{2}x^2 + C$

c. $x\ln^2 x - 2x\ln x - 2x + C$

d. $x\ln^2 x - 2x\ln x + 2x + C$

e. $x\ln^2 x - 2x\ln x + 4x + C$

7. Which of the following is the general partial fraction expansion of $\dfrac{4x^2+5}{(x-2)^2(x^2+9)^2}$?

a. $\dfrac{A}{(x-2)^2} + \dfrac{Bx+C}{(x^2+9)^2}$

b. $\dfrac{Ax+B}{(x-2)^2} + \dfrac{Cx+D}{(x^2+9)^2}$

c. $\dfrac{A}{(x-2)} + \dfrac{B}{(x-2)^2} + \dfrac{Cx+D}{(x^2+9)} + \dfrac{Ex+F}{(x^2+9)^2}$

d. $\dfrac{A}{(x-2)} + \dfrac{B}{(x-2)^2} + \dfrac{Cx}{(x^2+9)} + \dfrac{Dx}{(x^2+9)^2}$

e. $\dfrac{A}{(x-2)} + \dfrac{Bx+C}{(x-2)^2} + \dfrac{D}{(x^2+9)} + \dfrac{Ex+F}{(x^2+9)^2}$

8. The curve $x = y^3$ between $y = 0$ and $y = 1$ is rotated about the y-axis. Find the area of the resulting surface.

a. $\frac{\pi}{27}(10^{3/2} - 1)$

b. $48\pi(10^{3/2} - 1)$

c. $\frac{7}{27}\pi$

d. 336π

e. $48\pi 10^{3/2}$

9. Solve the differential equation $\frac{dy}{dx} = \frac{y}{x}$ with $y(1) = 3$. Then $y(3) =$

a. $\frac{1}{3}$

b. 1

c. 3

d. 6

e. 9

10. Solve the differential equation $x\frac{dy}{dx} = y + x^2$ with $y(1) = 2$. Then $y(2) =$

a. $\frac{1}{2}$

b. 2

c. $\frac{13}{6}$

d. 3

e. 6

11. A ball is dropped from 27 feet and bounces to $\frac{2}{3}$ of its previous height on each bounce. Find the total length travelled during an infinite number of bounces.

a. 54

b. 81

c. 108

d. 135

e. 162

12. Compute $\lim_{x \to 0} \frac{\sin x - x \cos x}{x^3}$

a. $\frac{1}{3}$

b. $\frac{1}{6}$

c. 0

d. ∞

e. $-\infty$

13. Compute $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{9^n (2n+1)!}$

a. $\frac{\sqrt{3}}{2}$

b. $\frac{3\sqrt{3}}{2}$

c. $\frac{\sqrt{3}}{6}$

d. $\frac{3}{2}$

e. 0

14. Find the center and radius of the sphere $x^2 - 4x + y^2 + z^2 + 6z + 4 = 0$

a. center: $(-2, 0, 3)$ radius: $R = 2$

b. center: $(2, 0, -3)$ radius: $R = 3$

c. center: $(-2, 0, 3)$ radius: $R = 3$

d. center: $(2, 0, -3)$ radius: $R = 9$

e. center: $(-2, 0, 3)$ radius: $R = 9$

Work Out (3 questions, Points indicated)

Show all you work.

15. (22 points) Consider the series $S = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{2^{n-1}}$.

a. (4) Determine whether the series is absolutely convergent, convergent but not absolutely convergent or divergent.

b. (1) Compute S_7, the 7^{th} partial sum for S. Do not simplify.

c. (3) Find a bound on the remainder $|R_7| = |S - S_7|$ when S_7 is used to approximate S. Name the theorem you used.

d. (1 Real easy) Find a power series $S(x)$ centered at 0 whose value at $x = \frac{-1}{2}$ is the given series S, i.e $S\left(\frac{-1}{2}\right) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{2^{n-1}}$.

#15 continued. Recall $S = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{2^{n-1}}$.

e. (10) Find the interval of convergence of the power series $S(x)$ from part (d). Give the radius and check the endpoints.

f. (2) Find a function $f(x)$ whose Maclaurin series is the power series $S(x)$ from part (d).

g. (1) Use $f(x)$ to find the sum of the series S.

16. (20 points) Let $f(x) = \ln(x)$.

a. (6) Find the Taylor series for $f(x)$ centered at $x = 4$.

b. (11) The Taylor series for $f(x)$ centered at $x = 3$ is $T(x) = \ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3^n n}(x-3)^n$.

Find the interval of convergence for the Taylor series centered at $x = 3$
Give the radius and check the endpoints.

c. (3) If the cubic Taylor polynomial centered at $x = 3$ is used to approximate $\ln(x)$ on the interval $[2,5]$, use the Taylor's Inequality to bound the error.

Taylor's Inequality:
Let $T_n(x)$ be the n^{th}-degree Taylor polynomial for $f(x)$ centered at $x = a$ and let $R_n(x) = f(x) - T_n(x)$ be the remainder. Then

$$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$$

provided $M \geq |f^{(n+1)}(c)|$ for all c between a and x.

17. (4 points) When a ball with mass, m, is dropped from a height, h, and falls under the forces of gravity with acceleration, g, and air resistance with drag coefficient, k, the altitude, $y(t)$, satisfies the differential equation,

$$m\frac{d^2y}{dt^2} = -mg - k\frac{dy}{dt}$$

The solution is

$$y(t) = h - \frac{m^2}{k^2}g\,e^{-\frac{kt}{m}} + \frac{m^2}{k^2}g - \frac{m}{k}g\,t$$

Verify that this solution reduces to the standard freefall formula (no air resistance) by taking the **limit** of the solution as k approaches 0. (DO NOT SOLVE ANY DIFFERENTIAL EQUATIONS.)

18. (4 points) The curve $x = y^2$ for $0 \le y \le \sqrt{2}$ is revolved around the x-axis. Find the area of the resulting surface.