

Name \_\_\_\_\_ Section \_\_\_\_\_  
 MATH 152H FINAL EXAM Spring 2016  
 Sections 201-202 P. Yasskin

Multiple Choice: (13 problems, 4 points each)

1-13	/52
14	/20
15	/20
16	/5
17	/5
18	/5
Total	/107

1.

**Average Value of a Function**

New Problem or Modify or Make Your Own Problem

Find the average value of the function  $f(x) = 2/9*x^2$  on the interval  $[a,b] = [0,3]$ .

- a.  $\frac{1}{6}$
- b.  $\frac{1}{3}$
- c.  $\frac{2}{3}$
- d.  $\frac{4}{3}$
- e. 2

2.

**Integrals Which are Improper at an Endpoint**

New Problem

Problem Statement:  
 Determine if the following improper integral is convergent or divergent.

$$\int_0^2 \frac{x}{(x-2)^3} dx$$

If convergent, compute it.  
 If divergent, determine if it is +infinity, -infinity, or neither.

- a. converges to  $\frac{1}{4}$
- b. converges to  $-\frac{1}{4}$
- c. diverges to  $-\infty$
- d. diverges to  $\infty$
- e. diverges but not to  $\pm\infty$

3.

**Integration By Parts**

Use integration by parts  
 to compute the integral:

$$J = \int_1^2 \frac{\ln(x)}{x^2} dx$$

- a.  $\frac{3 - \ln(2)}{2}$
- b.  $\frac{\ln(2) - 3}{2}$
- c.  $\frac{\ln(2) - 1}{2}$
- d.  $\frac{-\ln(2)}{2}$
- e.  $\frac{1 - \ln(2)}{2}$

4.

**Trigonometric Integrals**

Use a substitution  
 to compute the integral:

$$J = \int_0^{\frac{1}{3}\pi} \sec^4 \theta d\theta$$

- a.  $-2\sqrt{3}$
- b.  $2\sqrt{3}$
- c.  $-\frac{27}{5}$
- d.  $\frac{27}{5}$
- e.  $\frac{81}{5}\sqrt{3}$

5.

**Integration by Trigonometric Substitution**

New Integral

Goal: Evaluate the indefinite integral using a trigonometric substitution:

$$I = \int \frac{1}{x^2 \sqrt{(36x^2 - 1)}} dx$$

Simply identify the integral after the substitution.

- $\int \frac{36 \cos^3 \theta d\theta}{\sin \theta}$
- $\int \frac{\cos^3 \theta d\theta}{36 \sin \theta}$
- $\frac{1}{6} \int \cos \theta d\theta$
- $\int \cos \theta d\theta$
- $6 \int \cos \theta d\theta$

6.

**Partial Fractions: Finding Coefficients**

New Function  Include Completing the Square

Goal: Find the coefficients in the partial fraction expansion:

$$\frac{3x^2 - 3x - 2}{x^3(x+2)} = \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{A_3}{x^3} + \frac{A_4}{x+2}$$

Just find  $A_2$  and  $A_3$ .

- $A_2 = -\frac{1}{2} \quad A_3 = -2$
- $A_2 = \frac{1}{2} \quad A_3 = -2$
- $A_2 = -1 \quad A_3 = -1$
- $A_2 = 1 \quad A_3 = 1$
- $A_2 = 1 \quad A_3 = -1$

7.

**Volume By Slicing** \_ □ ×

New Problem or Modify or Make Your Own Problem Quit

Find the volume of the solid whose base is a semi-circle of radius 4 with the diameter edge parallel to the y axis, and whose cross sections perpendicular to the y direction are squares.

- a.  $\frac{8}{3}$
- b.  $\frac{128}{3}$
- c.  $\frac{256}{3}$
- d.  $8\pi$
- e.  $\frac{128}{3}\pi$

8.

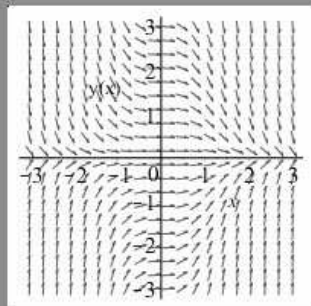
**Direction Fields** \_ □ ×

New Differential Equation Quit

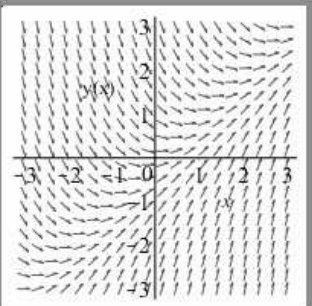
Problem Statement:

Find the direction field of the differential equation:  $\frac{d}{dx}y(x) = x - y$

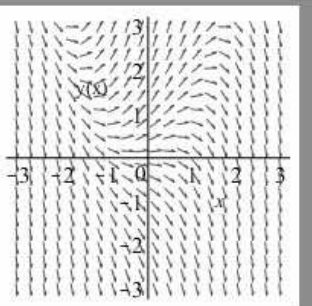
Select a Plot



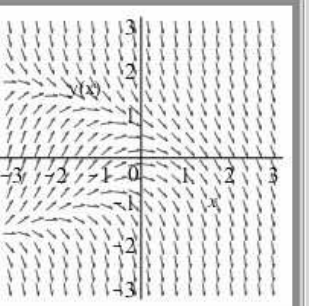
Plot # 1



Plot # 2



Plot # 3



Plot # 4

(a.)

(b.)

(c.)

(d.)

9.

**Arc Length of a Curve in 2D**

Functions of t  or

Find the arc length of the curve  $x = 2t^2$ ,  $y = 1/3t^3$ , between  $t = 0$  and  $t = 3$ .

- a.  $\frac{31}{3}$
- b.  $\frac{61}{3}$
- c.  $\frac{64}{3}$
- d.  $\frac{122}{3}$
- e.  $\frac{125}{3}$

10. The function  $y = \frac{x}{x^2 + 1}$  is a solution to which differential equation?

- a.  $\frac{dy}{dx} = \frac{y}{x^3 + x} + y^2$
- b.  $\frac{dy}{dx} = \frac{y}{x^3 + x} - y^2$
- c.  $\frac{dy}{dx} = \frac{y^2}{x^3 + x} + y^2$
- d.  $\frac{dy}{dx} = \frac{y^2}{x^3 + x} - y^2$
- e.  $\frac{dy}{dx} = -\frac{y}{x^3 + x} + y^2$

11.

**Geometric Series**

Goal: Compute the sum of the geometric series (if the sum exists).

$S = \sum_{n=2}^{\infty} a_n = \sum_{n=2}^{\infty} -1 \left( -\frac{2}{9} \right)^n$

- a.  $-\frac{9}{7}$
- b.  $-\frac{9}{11}$
- c.  $-\frac{4}{63}$
- d.  $-\frac{4}{99}$
- e. diverges

12.

**Computing Limits Using Maclaurin Series**

Goal: Use a Maclaurin Series to evaluate this limit:

$L = \lim_{x \rightarrow 0} \frac{\cos(x^2) - 1}{x^6}$

New Limit

- a. 0
- b.  $\frac{1}{2}$
- c.  $\frac{1}{3}$
- d.  $\frac{1}{24}$
- e. diverges

13.

**Separable Differential Equations**

New Differential Equation or Modify or Make Your Own Problem

Find a General Solution  Solve an Initial Value Problem

Find the solution  $y = F(x)$  of the differential equation  $\frac{dy}{dx} = -y^2 x^3$  satisfying the initial condition  $y(1) = 2$

- a.  $y = -\frac{4}{x^4} + 6$
- b.  $y = \frac{4}{x^4} - 2$
- c.  $y = \frac{4}{x^4 + 1}$
- d.  $y = \frac{4}{x^4 - 12}$
- e.  $y = \frac{4}{x^4} + \frac{3}{4}$

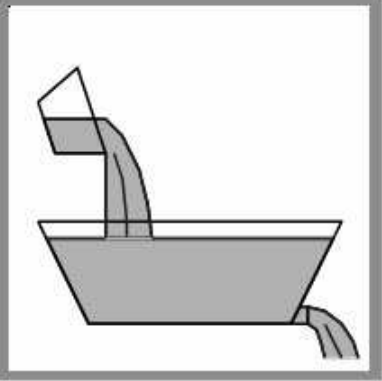
Work Out (5 questions, Points indicated. Show all you work.)

14. (20 points)

**Mixing Problem** [New Problem] [Quit]

Problem Statement:

A tank is initially filled with 650 L of sugar water with a concentration of 30 gm/L. A bucket is pouring sugar water of concentration 40 gm/L into the tank at a rate of 2 L/min. The solution is kept well mixed, and drains at a rate of 2 L/min. Find the total amount of sugar,  $S(t)$ , dissolved in the tank at time  $t$ . Then find the concentration of sugar in the tank at time  $t = 60$  min.



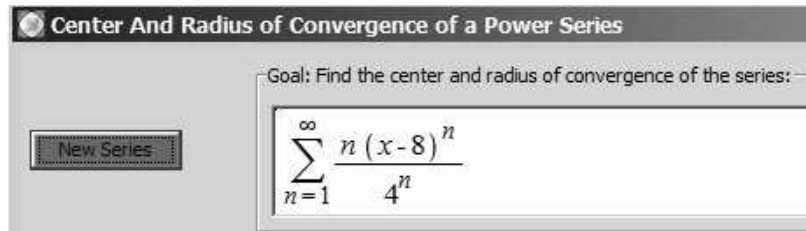
The diagram shows a trapezoidal tank with a flat top and a pointed bottom. On the left side, there is an inlet pipe where a stream of liquid is being poured into the tank. On the right side, there is an outlet pipe where a stream of liquid is being drained from the tank. The tank is partially filled with liquid, and the liquid level is indicated by a horizontal line.

a. (8 pts) Write the differential equation and initial condition for  $S(t)$ .

b. (9 pts) Solve the initial value problem for  $S(t)$ .

c. (3 pts) Find the concentration in the tank at  $t = 60$  min.

15. (20 points)



Also find the interval of convergence by checking the endpoints.

a. (2 pts) Identify the center:

$a =$  \_\_\_\_\_

b. (8 pts) Find the radius of convergence:

$R =$  \_\_\_\_\_

c. (8 pts) Check the endpoints:

d. (2 pts) Summarize the interval of convergence:

$I =$  \_\_\_\_\_



16. (5 points) Determine whether the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/3}}$  is absolutely convergent, convergent but not absolutely or divergent. Explain all tests you use.

17. (5 points) The series  $S = \sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$  converges by the Integral Test.

If it is approximated by its 100<sup>th</sup> partial sum  $S_{100}$ , compute the integral bound on the error in this approximation.

18. (5 points) Compute the sum of the series  $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n+1)! 3^{2n+1}}$ .