Name____________________________

| | |
|---|---|
| 1-13 | /52 |
| 14 | /15 |
| 15 | /20 |
| 16 | /15 |
| Total | /102 |

MATH 152H Exam 1 Spring 2016

Sections 201/202 (circle one) P. Yasskin

Multiple Choice: (4 points each. No part credit.)

**1**. Compute $\int_0^1 x^3 e^{(x^2+1)}\,dx$

**a**. $\frac{e}{2}$

**b**. $\frac{1}{2}(e^2-1)$

**c**. $\frac{e^2}{2}$

**d**. $\frac{1}{2}(e-1)$

**e**. $\frac{1}{2}(1-e)$

**2**. Compute $\int_0^{\pi/2} x\cos x\,dx$

**a**. $\pi-1$

**b**. $1-\pi$

**c**. $\frac{\pi}{2}$

**d**. $1+\frac{\pi}{2}$

**e**. $\frac{\pi}{2}-1$

**3.** Compute $\int_1^2 x^2 \ln x \, dx$

**a.** $\frac{8}{3}\ln 2 - 1$

**b.** $\frac{8}{3}\ln 2 - \frac{7}{9}$

**c.** $4\ln 2 - 1$

**d.** $4\ln 2 - 2$

**e.** $4\ln 2 - 3$

**4.** Compute $\int_0^\pi \sin^3\theta \, d\theta$

**a.** $\frac{1}{3}$

**b.** $\frac{2}{3}$

**c.** $1$

**d.** $\frac{4}{3}$

**e.** $2$

**5**. Compute $\int_0^{\pi/3} \sec^3\theta \tan^3\theta \, d\theta$

**a.** $0$

**b.** $\frac{56}{15}$

**c.** $\frac{58}{15}$

**d.** $\frac{128}{15}$

**e.** $\frac{136}{15}$

**6**. Compute $\int_0^{\pi/4} \cos 4\theta \cos 2\theta \, d\theta$

**a.** $1$

**b.** $\frac{1}{2}$

**c.** $\frac{1}{3}$

**d.** $\frac{1}{6}$

**e.** $\frac{1}{12}$

**7**. Find the area between, $y = x^2$ and $y = 3x$.

**a.** $\frac{81}{2}$

**b.** $\frac{27}{2}$

**c.** $\frac{9}{2}$

**d.** $\frac{1}{2}$

**e.** $\frac{10}{3}$

**8**. Find the average value of $f(x) = e^{3x}$ on the interval $[0, 2]$.

**a**. $\frac{1}{6}e^6$

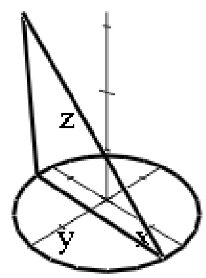
**b**. $\frac{1}{6}(e^6 - 1)$

**c**. $\frac{1}{3}e^6$

**d**. $\frac{1}{2}(1 + e^6)$

**e**. $(e^6 - 1)$

**9**. A solid has a base which is a circle of radius $2$ and has cross sections perpendicular to the $y$-axis which are isosceles right triangles with a leg on the base. Find the volume of the solid.




**a**. $\frac{64}{3}$

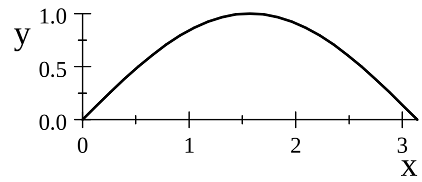
**b**. $\frac{128}{3}$

**c**. $\frac{16}{3}\pi$

**d**. $\frac{32}{3}\pi$

**e**. $\frac{32}{3}$

**10.** The region shown at the right is bounded by $y = \sin x$ and the $x$-axis. It is rotated about the line $y = -1$. Find the volume swept out.




**a.** $\dfrac{17\pi}{4}$

**b.** $\dfrac{9\pi}{2}$

**c.** $\dfrac{\pi^2}{2}$

**d.** $4\pi + 2\pi^2$

**e.** $4\pi + \dfrac{\pi^2}{2}$

**11.** The region in Problem 10 is rotated about the the line $x = -1$. Which formula gives the volume swept out?

**a.** $\int_0^\pi \pi(1 + \sin x)^2\,dx$

**b.** $\int_{-1}^\pi 2\pi x \sin x\,dx$

**c.** $\int_0^\pi \pi(x - 1)\sin x\,dx$

**d.** $\int_0^\pi 2\pi(x + 1)\sin x\,dx$

**e.** $\int_0^\pi \pi\left((1 + \sin x)^2 - 1\right)dx$

**12**. A certain spring is at rest when its mass is at $x = 3$.
It requires $24$ Joules of work to stretch it from $x = 3$ to $x = 7$ meters.
What is the force required to maintain the mass at $7$ meters?

**a**. $6$ Newtons

**b**. $12$ Newtons

**c**. $18$ Newtons

**d**. $24$ Newtons

**e**. $48$ Newtons

**13**. A $80$ kg cable which is $20$ meters long hangs down from the top of a building. A $5$ kg bag of sand is at the bottom of the cable. How much work is done to lift the sand and the cable to the top of the building? Give your answer as a multiple of the acceleration of gravity $g$.

**a**. $800g$

**b**. $900g$

**c**. $1600g$

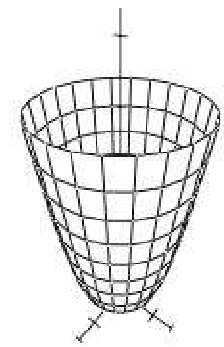
**d**. $1700g$

**e**. $2200g$

Work Out: (Points indicated. Part credit possible. Show all work.)

**14**. (15 points) Find the area between the cubic $y = x^3 - x^2$ and the line $y = 2x$.

**15**. (20 points) A water tower is made by rotating the curve $y = x^4$ about the $y$-axis, where $x$ and $y$ are in meters. If the tower is filled with water up to height $y = 25$ m, how much work is done to pump all the water out a faucet at height $30$ m?
Assume the acceleration of gravity is $g = 9.8$ m/sec$^2$ and the density of water is $\rho = 1000$ kg/m$^3$.
You may give your answer as a multiple of $\rho g \pi$.



**16.** $\left(15 \text{ points}\right)$ Consider the curve $y = f(x) = x^2$.

**a.** Find the tangent line at the generic point $x = p$. Call it $y = f_{\text{tan}}(x)$.

**b.** The region between $y = f(x)$ and $y = f_{\text{tan}}(x)$ for $0 \le x \le 1$ is rotated about the $y$-axis. Find the volume swept out. Call it $V(p)$.

**c.** Find the value of $p$ which minimizes the volume $V(p)$.

**d.** Repeat steps (a), (b) and (c) for the general concave up curve $y = f(x) = g(x)$.

Note: You can't do the integral $\int_0^1 g(x)x\,dx$. So leave it unevaluated.

(**a**)

(**b**)

(**c**)

**e.** What do you conclude?