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1-14	/56
15	/15
16	/15
17	/18
Total	/104

MATH 152H Exam 2 Spring 2016

Sections 201/202 (circle one) P. Yasskin

Multiple Choice: (4 points each. No part credit.)

1. Find the general partial fraction expansion of $f(x) = \dfrac{x^2-4}{(x^3+4x)(x^4-16)}$.

a. $\dfrac{A}{x} + \dfrac{Dx+E}{(x^2+4)^2}$

b. $\dfrac{A}{x} + \dfrac{Bx+C}{x^2-4} + \dfrac{Dx+E}{(x^2+4)^2}$

c. $\dfrac{A}{x} + \dfrac{Bx+C}{x^2-4} + \dfrac{Dx+E}{x^2+4}$

d. $\dfrac{A}{x} + \dfrac{Bx+C}{x^2+4} + \dfrac{Dx+E}{(x^2+4)^2}$

e. $\dfrac{A}{x} + \dfrac{Bx}{x^2-4} + \dfrac{Dx}{(x^2+4)^2}$

2. In the partial fraction expansion $\dfrac{x^3-4x-32}{x^4-16} = \dfrac{Ax+B}{x^2+4} + \dfrac{C}{x+2} + \dfrac{D}{x-2}$, which coefficient is INCORRECT?

a. $A = 1$

b. $B = 4$

c. $C = 1$

d. $D = 1$

e. All of the above are correct.

3. Compute $\int_{-1}^{4} \frac{1}{x^2}\,dx$

a. $\frac{5}{4}$

b. $-\frac{5}{4}$

c. diverges to $+\infty$

d. diverges to $-\infty$

e. diverges but not to $\pm\infty$

4. The integral $\int_{\pi}^{\infty} \frac{3+\sin(x)}{x}\,dx$ is

a. divergent by comparison with $\int_{\pi}^{\infty} \frac{2}{x}\,dx$.

b. divergent by comparison with $\int_{\pi}^{\infty} \frac{3}{x}\,dx$.

c. divergent by comparison with $\int_{\pi}^{\infty} \frac{4}{x}\,dx$.

d. convergent by comparison with $\int_{\pi}^{\infty} \frac{2}{x}\,dx$.

e. convergent by comparison with $\int_{\pi}^{\infty} \frac{4}{x}\,dx$.

5. Compute $\int_0^{4/3} \frac{1}{9x^2 + 16}\, dx$

a. $\frac{16}{9}$

b. $\frac{1}{9}$

c. $\frac{\pi}{12}$

d. $\frac{\pi}{24}$

e. $\frac{\pi}{48}$

6. Which of the following integrals results after performing an appropriate trig substitution for

$$\int_1^{4/3} \frac{1}{25x^2 - 16}\, dx$$

a. $\frac{1}{20} \int_{\text{arcsec}(5/3)}^{\text{arcsec}(5/4)} \csc\theta \, d\theta$

b. $\frac{1}{20} \int_{\text{arcsec}(5/4)}^{\text{arcsec}(5/3)} \csc\theta \, d\theta$

c. $\frac{5}{64} \int_{\text{arcsec}(5/3)}^{\text{arcsec}(5/4)} \sec\theta \, d\theta$

d. $\frac{5}{64} \int_{\text{arcsec}(5/4)}^{\text{arcsec}(5/3)} \sec\theta \, d\theta$

e. $\frac{1}{16} \int_{\text{arcsec}(5/4)}^{\text{arcsec}(5/3)} \cot^2\theta \, d\theta$

7. Compute $\int_0^{3/5} \frac{1}{25x^2 - 16}\,dx$

a. $-\frac{1}{20}$

b. $\frac{1}{20}$

c. $-\frac{1}{40}\ln 7$

d. $\frac{1}{20} - \frac{1}{40}\ln 7$

e. $\frac{1}{40}\ln 7 - \frac{1}{20}$

8. Find the length of the curve $x = 2t^2$, and $y = t^3$ for $0 \le t \le 1$.

a. 732

b. 108

c. 7

d. $\frac{61}{27}$

e. $\frac{1}{3}$

9. The curve $y = \sin x$ for $0 \le x \le \pi$ is rotated about the x-axis.
Which integral gives the area of the resulting surface?

a. $\int_0^{\pi} 2\pi x \sqrt{1 + \cos^2 x}\, dx$

b. $\int_0^{\pi} 2\pi \sin x \sqrt{1 + \cos^2 x}\, dx$

c. $\int_0^{\pi} 2\pi \cos x \sqrt{1 + \sin^2 x}\, dx$

d. $\int_0^{\pi} \pi \cos x \sqrt{1 + \sin^2 x}\, dx$

e. $\int_0^{\pi} \pi \sin x\, dx$

10. The curve $y = \sin x$ for $0 \le x \le \pi$ is rotated about the y-axis.
Which integral gives the area of the resulting surface?

a. $\int_0^{\pi} 2\pi x \sqrt{1 + \cos^2 x}\, dx$

b. $\int_0^{\pi} 2\pi \sin x \sqrt{1 + \cos^2 x}\, dx$

c. $\int_0^{\pi} 2\pi \cos x \sqrt{1 + \sin^2 x}\, dx$

d. $\int_0^{\pi} \pi \cos x \sqrt{1 + \sin^2 x}\, dx$

e. $\int_0^{\pi} \pi \sin x\, dx$

11. Consider the sequence $a_n = \arctan\left(\dfrac{3n-6}{\sqrt{3}\,n-6}\right)$. Compute $\lim\limits_{n\to\infty} a_n$.

a. π

b. $\dfrac{\pi}{2}$

c. $\dfrac{\pi}{3}$

d. $\dfrac{\pi}{4}$

e. $\dfrac{\pi}{6}$

12. $\displaystyle\sum_{n=1}^{\infty} 3\frac{2^{2n}}{3^n} =$

a. -12

b. 12

c. -9

d. 9

e. diverges

13. $\displaystyle\sum_{n=1}^{\infty} 3\frac{2^{n}}{3^n} =$

a. 3

b. 6

c. 9

d. 12

e. diverges

14. $\sum_{n=3}^{\infty} \ln\left(\frac{n^2}{n^2-1}\right) =$

a. $\ln \frac{3}{4}$

b. $\ln \frac{3}{2}$

c. $\ln 3$

d. $\ln \frac{1}{4}$

e. $\ln \frac{1}{2}$

Work Out: (Points indicated. Part credit possible. Show all work.)

15. $\left(15 \text{ points}\right)$ Compute $\int \frac{3(x-3)^2}{x^4-81}\,dx$. Simplify to a single $\ln$.

16. (15 points) Compute $\int_{1}^{4/3} \frac{1}{\sqrt{25x^2 - 16}}\, dx$. Simplify to a single $\ln$.

17. (18 points) Consider the sequence defined recurrsively by

$$a_1 = 2 \qquad \text{and} \qquad a_{n+1} = 5 - \frac{4}{a_n}$$

a. (6 pts) Use mathematical induction to show a_n is positive and increasing for all n, i.e. $a_{n+1} \geq a_n > 0$.

i. To get started, show $a_2 \geq a_1 > 0$:

ii. Assume $a_{k+1} \geq a_k > 0$ and prove $a_{k+2} \geq a_{k+1} > 0$:

Therefore, $a_{n+1} \geq a_n > 0$ for all n.

b. (6 pts) Use mathematical induction to show a_n is bounded above by 8.

i. To get started, show $a_1 < 8$ and $a_2 < 8$:

ii. Assume $a_k < 8$ and prove $a_{k+1} < 8$:

Therefore, $a_n < 8$ for all n.

c. (6 pts) What do you conclude about $\lim_{n\to\infty} a_n$? Find $\lim_{n\to\infty} a_n$.