Name_____

1-13	/52
14	/20
15	/16
16	/18
Total	/106

MATH 152H Exam 3 Spring 2016

Sections 201/202 (circle one) P. Yasskin

Multiple Choice: (4 points each. No part credit.)

1. The series $S = \sum_{n=1}^{\infty} (3^{1/n} - 3^{1/(n+1)})$ is

a. absolutely convergent.

b. conditionally convergent.

c. divergent by the n^{th} Term Divergence Test

d. divergent by the Alternating Series Test.

e. divergent because it is the difference between two p-series with $p = \frac{1}{n} < 1$ and $p = \frac{1}{n+1} < 1$.

2. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is

a. convergent and absolutely convergent.

b. absolutely convergent but not convergent.

c. convergent but not absolutely convergent.

d. divergent.

3. If $f(x) = \sin(x^2)$, compute $f^{(6)}(0)$.

a. $-6 \cdot 6!$

b. $-6 \cdot 3!$

c. $6 \cdot 3!$

d. $-5!$

e. $5!$

4. Compute $\sum_{n=0}^{\infty} \frac{(-2)^n}{n!}$ HINT: Think about a known Maclaurin series.

a. $\frac{1}{e^2}$

b. $\frac{-1}{e^2}$

c. $\frac{1}{2}$

d. $-e^2$

e. The series diverges.

5. Compute $\lim_{x \to 0} \frac{\cos(x^3) - 1 + \frac{1}{2}x^6}{x^{12}}$

a. $-\frac{1}{24}$

b. $-\frac{1}{4}$

c. $\frac{1}{4}$

d. $\frac{1}{6}$

e. $\frac{1}{24}$

6. The series $S = \sum_{n=0}^{\infty} \frac{3^n}{1+4^n}$ satisfies

a. $S = 0$

b. $0 < S < 4$

c. $S = 4$

d. $S > 4$

e. The series diverges.

7. Find the equation of the sphere whose diameter has endpoints at $(2,1,6)$ and $(4,-3,8)$.

a. $(x+6)^2 + (y-2)^2 + (z+14)^2 = 6$

b. $(x+3)^2 + (y-1)^2 + (z+7)^2 = 24$

c. $(x-3)^2 + (y+1)^2 + (z-7)^2 = 6$

d. $(x-3)^2 + (y+1)^2 + (z-7)^2 = 24$

e. $(x-6)^2 + (y+2)^2 + (z-14)^2 = 24$

8. A triangle has vertices $A = (1,2,3)$, $B = (2,3,3)$ and $C = (1,3,2)$. Find the angle at A.

a. $0°$

b. $30°$

c. $45°$

d. $60°$

e. $90°$

9. The 3^{rd} degree Taylor polynomial for $f(x) = \frac{1}{\sqrt{x}}$ at $x = 4$ is

$$T_3(x) = f(4) + f'(4)(x-4) + \frac{1}{2}f''(4)(x-4)^2 + \frac{1}{6}f'''(4)(x-4)^3$$
$$= \frac{1}{2} - \frac{1}{16}(x-4) + \frac{3}{256}(x-4)^2 - \frac{5}{2048}(x-4)^3$$

and the third derivative is $f'''(x) = \frac{-15}{8}x^{-7/2}$.

If the 2^{nd} degree Taylor polynomial $T_2(x)$ is used to approximate $f(6) = \frac{1}{\sqrt{6}}$ we get

$$T_2(6) = \frac{1}{2} - \frac{1}{16}(6-4) + \frac{3}{256}(6-4)^2 = \frac{27}{64} = .4219$$

Which of the following is the best bound on the error in this approximation?

a. $|E_2| < \frac{5}{128} \approx .0391$
b. $|E_2| < \frac{5}{256} \approx .0195$
c. $|E_2| < \frac{15}{128} \approx .1172$
d. $|E_2| < \frac{15}{256} \approx .0586$
e. $|E_2| < \frac{15}{512} \approx .0293$

10. Suppose the function $y = f(x)$ is the solution of $\frac{dy}{dx} = x^2 - y^2$ satisfying the initial condition $f(1) = 2$. Find $f'(1)$.

a. -1
b. -2
c. -3
d. 1
e. 2

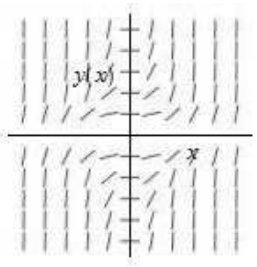
11. Find the solution, $y = f(x)$, of the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$ satisfying the initial condition $f(0) = 3$. What is $f(3)$?

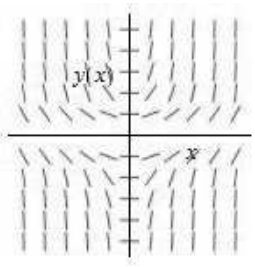
a. $3\sqrt{2}$
b. $3\sqrt[3]{2}$
c. $\sqrt[3]{36}$
d. 6
e. 18

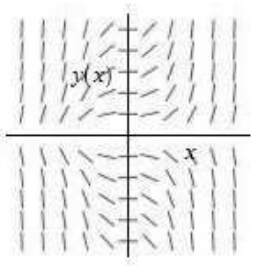
12. Find the integrating factor for the linear differential equation $x^4 \frac{dy}{dx} = 4x^3y + x^2$.

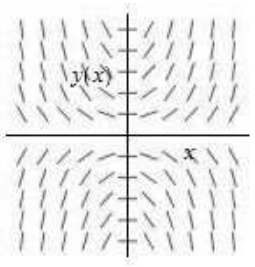
a. x^4
b. $4\ln x$
c. $-4\ln x$
d. $e^{-4/x}$
e. x^{-4}

13. Which of the following is the direction field of the differential equation $\frac{dy}{dx} = x^2y$?

a. 

b. 

c. 

d. 

Work Out: (Points indicated. Part credit possible. Show all work.)

14. (20 points) For each series, determine if it is convergent or divergent.
Be sure to identify the Convergence Test(s) and check out their hypotheses.

a. (6 pts) $\sum_{n=2}^{\infty} n e^{(-n^2)}$

b. (6 pts) $\sum_{n=2}^{\infty} \frac{n^5 - 3}{n^6 + 1}$

c. (8 pts) $\sum_{n=2}^{\infty} \frac{\sin n}{n^2 + 1}$

15. (16 points) Find the radius and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n3^n}(x-4)^n$.

Be sure to identify the Convergence Test(s) and check out their hypotheses.

16. (18 points) A salt water fish tank contains 20 liters of water with 700 grams of salt. In order to reduce the salt concentration, you pour in salt water with a concentration of 15 grams of salt per liter at 2 liters per minute. You keep the tank well mixed and drain the mixture at 2 liters per minute. Let $S(t)$ be the amount of salt (in grams) in the tank at time t (in minutes).

a. (6 pts) Write the differential equation and initial condition for $S(t)$.

b. (9 pts) Solve the initial value problem for $S(t)$.

c. (3 pts) After how many minutes will the amount of salt in the tank drop to 400 grams?