Name____________________ ID___________

1-8	/40
9	/10
10	/10
11	/20
12	/10
13	/10
Total	/100

MATH 171 Exam 1 Spring 2004

Sections 502 Solutions P. Yasskin

On the front of the Blue Book, on the Scantron and on this sheet
write your Name, your University ID and "Exam 1."
On the front of the Blue Book copy the Grading Grid shown at the right.
Enter your Multiple Choice answers on the Scantron
and CIRCLE them on this sheet.

Multiple Choice: (5 points each. No part credit.)

1. Compute: $\lim\limits_{x\to5}\dfrac{x-5}{x^2-25}$

a. $\dfrac{1}{10}$ correctchoice

b. $\dfrac{1}{5}$

c. 0

d. 5

e. Does Not Exist

$$\lim_{x\to5}\frac{x-5}{x^2-25}=\lim_{x\to5}\frac{x-5}{(x-5)(x+5)}=\lim_{x\to5}\frac{1}{x+5}=\frac{1}{10}$$

2. Compute: $\lim\limits_{x\to2}\dfrac{(x+1)^2-(x-1)^2-8}{x-2}$

a. 1

b. 2

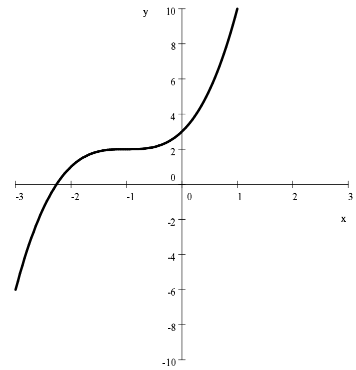
c. 4 correctchoice

d. 8

e. Does Not Exist

$$\lim_{x\to2}\frac{(x+1)^2-(x-1)^2-8}{x-2}=\lim_{x\to2}\frac{(x^2+2x+1)-(x^2-2x+1)-8}{x-2}=\lim_{x\to2}\frac{4x-8}{x-2}=4$$

3. Which of the following is the function whose graph is →→→



a. $f(x) = (x-2)^3 - 1$

b. $f(x) = (x-1)^3 + 2$

c. $f(x) = (x+1)^3 + 2$ correctchoice

d. $f(x) = (x+1)^3 - 2$

e. $f(x) = (x+2)^3 + 1$

x^3 is shifted left by 1 and up by 2. So $f(x) = (x+1)^3 + 2$

4. A triangle has vertices $A = (-3, 13)$, $B = (2, 1)$ and $C = (6, 4)$. Find $\cos\theta$ where θ is the angle at vertex B.

a. $\dfrac{17}{\sqrt{13}\sqrt{178}}$

b. $\dfrac{16}{845}$

c. $\dfrac{845}{16}$

d. $\dfrac{16}{65}$ correctchoice

e. $\dfrac{65}{16}$

$$\overrightarrow{BA} = A - B = (-3, 13) - (2, 1) = (-5, 12) \qquad \left|\overrightarrow{BA}\right| = \sqrt{25 + 144} = 13$$

$$\overrightarrow{BC} = C - B = (6, 4) - (2, 1) = (4, 3) \qquad \left|\overrightarrow{BC}\right| = \sqrt{16 + 9} = 5$$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = -20 + 36 = 16 \qquad \cos\theta = \frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{\left|\overrightarrow{BA}\right|\left|\overrightarrow{BC}\right|} = \frac{16}{5 \cdot 13} = \frac{16}{65}$$

5. A wagon is pulled along the ground by exerting a 4 Newton force along the handle which makes a $30°$ angle with the horizontal. How much work is done in pulling the wagon 5 meters?

a. 10 Joules

b. $10\sqrt{3}$ Joules correctchoice

c. 5 Joules

d. $5\sqrt{3}$ Joules

e. $20\sqrt{3}$ Joules

$$W = \vec{F} \cdot \vec{D} = \left|\vec{F}\right|\left|\vec{D}\right| \cos\theta = 4 \cdot 5 \cdot \cos 30° = 20\frac{\sqrt{3}}{2} = 10\sqrt{3}$$

6. Find the parametric equations of the line through the points $A = (-3, 13)$ and $B = (2, 1)$.

a. $x = -3 + 5t, \quad y = 13 - 12t$ correctchoice

b. $x = 5 - 3t, \quad y = -12 + 13t$

c. $x = -3 + 2t, \quad y = 13 + t$

d. $x = 2 - 3t, \quad y = 1 + 13t$

e. $x = 5 + 2t, \quad y = -12 + t$

$\overrightarrow{AB} = B - A = (2, 1) - (-3, 13) = (5, -12)$
$X = A + t\overrightarrow{AB} \qquad (x, y) = (-3, 13) + t(5, -12) = (-3 + 5t, 13 - 12t)$

7. Which of the following parametric curves is the parabola $x = 2 + y^2$?

a. $x = 2 - t, \quad y = t^2$

b. $x = t^2, \quad y = 2 + t$

c. $x = 2 + t, \quad y = t^2$

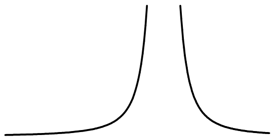
d. $x = t, \quad y = 2 + t^2$

e. $x = 2 + t^2, \quad y = t$ correctchoice

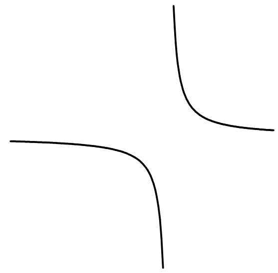
Eliminate t from each equation:
a) $x = 2 - \sqrt{y}$ b) $x = (y - 2)^2$ c) $x = 2 + \sqrt{y}$ d) $y = 2 + x^2$ e) $x = 2 + y^2$

8. Near the point $x = 3$, the graph of the function $f(x) = \dfrac{x^2 - 5x + 6}{x^2 - 6x + 9}$ looks qualitatively like

a.

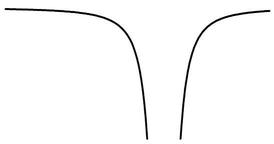


b.

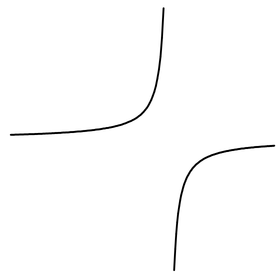


correctchoice

c.



d.



$$\frac{x^2 - 5x + 6}{x^2 - 6x + 9} = \frac{(x-3)(x-2)}{(x-3)^2} = \frac{x-2}{x-3}$$

$\lim\limits_{x \to 3^-} \dfrac{x-2}{x-3} = \text{“}\dfrac{1^-}{0^-}\text{”} = -\infty$ $\qquad$ $\lim\limits_{x \to 3^+} \dfrac{x-2}{x-3} = \text{“}\dfrac{1^+}{0^+}\text{”} = +\infty$ $\qquad$ (b)

Work Out: (Points indicated. Part credit possible.)

Start each problem on a new page of the Blue Book. Number the problem. Show all work.

9. $(10$ points$)$ State the meaning of the equation $\lim\limits_{x \to 5} (3x - 4) = 11$ and then prove it. Be sure to distinguish between your Definition, your Scratch work and your Proof.

Definition: $\lim\limits_{x \to 5} (3x - 4) = 11$ means:

For all $\varepsilon > 0$ there is a $\delta > 0$ such that if $0 < |x - 5| < \delta$ then $|(3x - 4) - 11| < \varepsilon$.

Scratch: $|(3x - 4) - 11| < \varepsilon$ $\quad |3x - 15| < \varepsilon$ $\quad |x - 5| < \dfrac{\varepsilon}{3}$ $\quad \delta = \dfrac{\varepsilon}{3}$

Proof: Given $\varepsilon > 0$ let $\delta = \dfrac{\varepsilon}{3}$. If $0 < |x - 5| < \delta = \dfrac{\varepsilon}{3}$, then $|3x - 15| < \varepsilon$ or $|(3x - 4) - 11| < \varepsilon$.

10. $(10$ points$)$ Find an interval of width 1 in which the equation $x^3 - x = 1$ is guaranteed to have a solution. Be sure to name the theorem you used and explain why it applies.

Let $f(x) = x^3 - x$. Then $f(0) = 0$, $f(1) = 0$ and $f(2) = 6$. Since f is continuous and $0 \le 1 \le 6$, the Intermediate Value Theorem guarantees that there is a number c in $[1,2]$ where $f(c) = 1$, i.e. $c^3 - c = 1$. So $x = c$ is a solution and the interval is $[1,2]$.

11. (20 points) A body is moving so that its position at time t is $x(t) = \sqrt{t+2}$.

a. What is the average velocity between $t = 2$ and $t = 7$?

$$v_{ave} = \frac{x(7) - x(2)}{7 - 2} = \frac{\sqrt{7+2} - \sqrt{2+2}}{5} = \frac{3-2}{5} = \frac{1}{5}$$

b. What is the average velocity between $t = 2$ and $t = 2 + h$?

$$v_{ave} = \frac{x(2+h) - x(2)}{(2+h) - 2} = \frac{\sqrt{2+h+2} - \sqrt{2+2}}{h} = \frac{\sqrt{4+h} - 2}{h}$$

c. What is the instantaneous velocity at $t = 2$?

$$v = \lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h} = \lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h} \cdot \frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2} = \lim_{h \to 0} \frac{(4+h) - 4}{h\left(\sqrt{4+h} + 2\right)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{4+h} + 2} = \frac{1}{4}$$

12. (10 points) Compute the derivative of $f(x) = \frac{1}{x}$ from the limit definition of the derivative.

HINTS: $\frac{a-b}{c} = \frac{1}{c}(a-b)$ Put everything over a common denominator.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{1}{h}\left(\frac{1}{x+h} - \frac{1}{x}\right) = \lim_{h \to 0} \frac{1}{h}\left(\frac{x - (x+h)}{(x+h)x}\right)$$

$$= \lim_{h \to 0} \frac{-1}{(x+h)x} = \frac{-1}{x^2}$$

13. (10 points) Find the horizontal asymptotes as $x \to \infty$ and as $x \to -\infty$ of the function $f(x) = \frac{\sqrt{x^2+4x} - \sqrt{x^2+2x}}{2}$. Be sure to state your two answers in concluding sentences, identifying which asymptote is which.

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{\sqrt{x^2+4x} - \sqrt{x^2+2x}}{2} \cdot \frac{\sqrt{x^2+4x} + \sqrt{x^2+2x}}{\sqrt{x^2+4x} + \sqrt{x^2+2x}} = \lim_{x \to \infty} \frac{(x^2+4x) - (x^2+2x)}{2\left(\sqrt{x^2+4x} + \sqrt{x^2+2x}\right)}$$

$$= \lim_{x \to \infty} \frac{(x^2+4x) - (x^2+2x)}{2\left(\sqrt{x^2+4x} + \sqrt{x^2+2x}\right)} = \lim_{x \to \infty} \frac{2x}{2\left(\sqrt{x^2+4x} + \sqrt{x^2+2x}\right)} \cdot \frac{\frac{1}{x}}{\frac{1}{\sqrt{x^2}}}$$

$$= \lim_{x \to \infty} \frac{1}{\sqrt{1 + \frac{4}{x}} + \sqrt{1 + \frac{2}{x}}} = \frac{1}{2}$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{2x}{2\left(\sqrt{x^2+4x} + \sqrt{x^2+2x}\right)} \cdot \frac{\frac{1}{x}}{\frac{-1}{\sqrt{x^2}}} = \lim_{x \to -\infty} \frac{-1}{\sqrt{1 + \frac{4}{x}} + \sqrt{1 + \frac{2}{x}}} = \frac{-1}{2}$$

The horizontal asymptote as $x \to \infty$ is $y = \frac{1}{2}$. The horizontal asymptote as $x \to -\infty$ is $y = \frac{-1}{2}$.