Name____________________ ID___________

1-10	/40
11	/15
12	/15
13	/15
14	/15
Total	/100

MATH 171 Exam 2 Spring 2004

Sections 502 Solutions P. Yasskin

On the front of the Blue Book, on the Scantron and on this sheet
write your Name, your University ID and "Exam 2."
On the front of the Blue Book copy the Grading Grid shown at the right.
Enter your Multiple Choice answers on the Scantron
and CIRCLE them on this sheet.

Multiple Choice: (4 points each. No part credit.)

1. If $f(x) = 3x^3 - 7x^2 + 2$ then $f'(x) =$

a. $6x^2 - 7x$

b. $6x^2 - 7x + 2$

c. $3x^2 - 7x + 2$

d. $9x^2 - 14x$ correctchoice

e. $9x^2 - 14x + 2$

By the sum rule, constant multiple rule, power rule and constant rule $f'(x) = 9x^2 - 14x$.

2. If $f(x) = \frac{x}{x+1}$ then $\frac{df}{dx} =$

a. $\frac{1}{(x+1)^2}$ correctchoice

b. $\frac{-1}{(x+1)^2}$

c. $\frac{2x+1}{(x+1)^2}$

d. $\frac{2x-1}{x+1}$

e. $\frac{2x+1}{x+1}$

By the quotient rule, $\frac{df}{dx} = \frac{(x+1)(1) - (x)(1)}{(x+1)^2} = \frac{1}{(x+1)^2}$.

3. If $f(x) = x^2 \cos x$ then $f'\left(\frac{\pi}{2}\right) =$

a. $\frac{\pi}{2}$

b. π

c. $-\pi$

d. $\frac{\pi^2}{4}$

e. $\frac{-\pi^2}{4}$ correctchoice

By the product rule, $f'(x) = x^2 \frac{d}{dx} \cos x + \cos x \frac{d}{dx} x^2 = -x^2 \sin x + 2x \cos x.$
So $f'\left(\frac{\pi}{2}\right) = -\left(\frac{\pi}{2}\right)^2 \sin\left(\frac{\pi}{2}\right) + 2\left(\frac{\pi}{2}\right) \cos\left(\frac{\pi}{2}\right) = \frac{-\pi^2}{4}$ since $\sin\left(\frac{\pi}{2}\right) = 1$ and $\cos\left(\frac{\pi}{2}\right) = 0.$

4. If $f(x) = e^x + x^e$ then $\frac{df}{dx} =$

a. $x e^{x-1} + e x^{e-1}$

b. $e^x + x^e$

c. $e^x + e x^{e-1}$ correctchoice

d. $e^x + (\ln x) x^e$

e. $e^x + \frac{x^e}{\ln x}$

$\frac{df}{dx} = \frac{d}{dx} e^x + \frac{d}{dx} x^e = e^x + e x^{e-1}$

5. Find the equation of the line tangent to the graph of $y = x^{1/2}$ at $x = 4$. What is its y-intercept?

a. 0

b. 1 correctchoice

c. 2

d. 3

e. $\frac{1}{4}$

$f(x) = x^{1/2}$ $\quad f'(x) = \frac{1}{2} x^{-1/2}$ $\quad f(4) = 4^{1/2} = 2$ $\quad f'(4) = \frac{1}{2} 4^{-1/2} = \frac{1}{4}$
So the tangent line is $y = f(4) + f'(4)(x-4) = 2 + \frac{1}{4}(x-4) = \frac{1}{4}x + 1$ and the y-intercept is $b = 1.$

6. Compute $\frac{d}{dx}\ln(x^2+3)$.

a. $\frac{2x+3}{x^2+3}$

b. $\frac{2x}{(x^2+3)^2}$

c. $\frac{1}{x^2+3}$

d. $\frac{2x}{x^2+3}$ correctchoice

e. $\frac{1}{\ln x}\frac{1}{x^2+3}$

By the chain rule $\frac{d}{dx}\ln(x^2+3) = \frac{1}{x^2+3}\frac{d}{dx}(x^2+3) = \frac{2x}{x^2+3}$

7. Find the slope of the tangent line to the graph of $x^3+3xy+y^3 = 15$ at $(1,2)$.

a. $\frac{-4}{5}$

b. $\frac{-3}{5}$ correctchoice

c. $\frac{-1}{5}$

d. $\frac{3}{5}$

e. $\frac{4}{5}$

We implicitly differentiate by applying $\frac{d}{dx}$ to both sides using the product and chain rules:

$$3x^2+3y+3x\frac{dy}{dx}+3y^2\frac{dy}{dx} = 0 \qquad \frac{dy}{dx} = \frac{-3x^2-3y}{3x+3y^2} = \frac{-x^2-y}{x+y^2} \qquad \left.\frac{dy}{dx}\right|_{(1,2)} = \frac{-1^2-2}{1+2^2} = \frac{-3}{5}$$

8. If the position of a rocket is $\vec{r}(t) = (t^3-9t, \sin t)$, find its acceleration at $t = \frac{\pi}{2}$.

a. $\left(\frac{3\pi^2}{4}-9, 0\right)$

b. $(3\pi-9, 1)$

c. $(3\pi, 1)$

d. $(3\pi-9, -1)$

e. $(3\pi, -1)$ correctchoice

$$\vec{v}(t) = (3t^2-9, \cos t) \qquad \vec{a}(t) = (6t, -\sin t) \qquad \vec{a}\left(\frac{\pi}{2}\right) = \left(6\frac{\pi}{2}, -\sin\frac{\pi}{2}\right) = (3\pi, -1)$$

9. Use Newton's method to find an approximate solution to $x^3 - 2x - 3 = 0$. If $x_0 = 2$, then

a. $x_1 = 1.90$ correctchoice

b. $x_1 = 1.99$

c. $x_1 = 2.01$

d. $x_1 = 2.1$

e. $x_1 = 2.2$

$f(x) = x^3 - 2x - 3 \qquad f'(x) = 3x^2 - 2 \qquad x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 2 - \dfrac{f(2)}{f'(2)} = 2 - \dfrac{1}{10} = 1.9$

10. Suppose $g(x)$ is the inverse function of $f(x)$, which satisfies

$$f(2) = 3 \qquad f(3) = 4 \qquad f'(2) = 5 \qquad f'(3) = 6 \qquad f'(4) = 7$$

Then $g'(3) =$

a. $-\frac{1}{6}$

b. $\frac{1}{4}$

c. $\frac{1}{5}$ correctchoice

d. $\frac{1}{6}$

e. $\frac{1}{7}$

We have $g(3) = 2$. So the tangent line to $y = g(x)$ at $x = 3$ is the mirror image of the tangent line to $y = f(x)$ at $x = 2$. Thus,

$g'(x) = \dfrac{1}{f'(g(x))}$ or $g'(3) = \dfrac{1}{f'(g(3))} = \dfrac{1}{f'(2)} = \dfrac{1}{5}$.

Work Out: (Points indicated. Part credit possible.)

Start each problem on a new page of the Blue Book. Number the problem. Show all work.

11. (15 points) As Duke Skywater flies across the galaxy, he measures the density of the polaron field $P(t)$ as a function of time. After $t = 3$ hours of travelling, he measures the polaron density is $P(3) = 132$ wookies/meter^3 and is increasing by $\frac{dP}{dt}(3) = 4$ wookies/meter^3/hour. Approximately what will the polaron density be at $t = 5$ hours?

By the linear approximation, $P(t) \approx P(3) + P'(3)(t-3) = 132 + 4(t-3)$.

So $P(5) \approx 132 + 4(5-3) = 140$ wookies/meter^3.

12. (15 points) A 10 ft ladder is leaning against a vertical wall but sliding down. The base of the ladder is currently 6 ft from the wall and moving away at 2 ft/sec. How fast is the top of the ladder moving down the wall? You must explain your solution using sentences.

Let x be the distance from the base of the ladder to the wall and let y be the height of the top of the ladder. Then by the Pythagorean Theorem, $x^2 + y^2 = 10^2$, where x and y are functions of the time t.

Applying $\frac{d}{dt}$ to both sides, we get: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

We solve for $\frac{dy}{dt}$: $\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$

Finally we plug in the current values: $x = 6$, $\frac{dx}{dt} = 2$, $y = \sqrt{100 - x^2} = \sqrt{100 - 36} = 8$

to get: $\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt} = \frac{-6}{8}2 = \frac{-3}{2}$

So the ladder is sliding down the wall at $\frac{3}{2}$ ft/sec.

13. (15 points) Given that $\frac{d}{dx}\tan x = \sec^2 x$, derive $\frac{d}{dx}\arctan x$.

The answer must be a function of x, not y. Here are some identities:

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

$y = \arctan x$ means $x = \tan y$ Apply $\frac{d}{dx}$ to both sides and use chain rule:

$$1 = \sec^2 y\frac{dy}{dx} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{\tan^2 y + 1} = \frac{1}{x^2 + 1}$$

14. (15 points) In 3 days a 100 kg sample of radon-222 decays into 58 kg.

(Round answers to 3 digits.)

a. Find a formula, $A(t)$, for the amount of radon-222 after t days.

$$A(t) = 100e^{-kt} \qquad A(3) = 100e^{-3k} = 58 \qquad e^{-3k} = .58 \qquad k = \frac{-\ln(.58)}{3} \approx .182$$

$$A(t) = 100e^{-.182t}$$

b. What is the half-life of radon-222?

$$50 = 100e^{-.182t} \qquad e^{-.182t} = .5 \qquad t = \frac{-\ln .5}{.182} \approx 3.81$$

c. How many days will it take for a 100 kg sample of radon-222 to decay into 10 kg?

$$10 = 100e^{-.182t} \qquad e^{-.182t} = .1 \qquad t = \frac{-\ln .1}{.182} \approx 12.7$$