Name____________________ ID____________________

1-10	/60
11	/25
12	/15

MATH 172 EXAM 3 Spring 1999

Section 504 P. Yasskin

Multiple Choice: (6 points each)

1. Compute $\lim\limits_{n\to\infty} \dfrac{\ln n}{n}$

a. $-\infty$
b. -1
c. 0
d. e
e. ∞

2. The series $\sum\limits_{n=1}^{\infty} \dfrac{\ln n}{n}$ is

a. Divergent by the n^{th} Term Divergence Test
b. Divergent by the Integral Test
c. Convergent by the Integral Test
d. Divergent by the Ratio Test
e. Convergent by the Ratio Test

3. A convergent sequence is recursively defined by $a_1 = 1$ and $a_{n+1} = \dfrac{3 - a_n}{2 + a_n}$.
Find $\lim\limits_{n\to\infty} a_n$.

a. $\dfrac{19}{24}$
b. $\dfrac{\sqrt{21} - 3}{2}$
c. $\dfrac{\pi}{4}$
d. $\dfrac{3}{2}$
e. $\dfrac{2}{3}$

4. The sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 3^{n+1}}{4^n}$ is:

a. nonexistant since its partial sums oscillate
b. $-\frac{3}{4}$
c. $\frac{9}{7}$
d. nonexistant by the n^{th} Term Divergence Test
e. $\frac{3}{16}$

5. Compute $\sum_{k=2}^{\infty} \frac{3}{(n-1)(n+1)}$. (HINTS: partial fractions,telescoping sum)

a. $\frac{9}{4}$
b. $\frac{3}{2}$
c. 1
d. $\frac{1}{2}$
e. ∞

6. The series $\sum_{n=1}^{\infty} (-1)^n \frac{3^n n^2}{n!}$ is

a. Absolutely convergent
b. Conditionally convergent
c. Divergent

7. The series $\sum_{n=1}^{\infty} (-1)^n \frac{(n+3) 2^{2n}}{3^{n+100}}$ is

a. Absolutely convergent
b. Conditionally convergent
c. Divergent

8. Given that $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$, compute $\lim_{x \to 0} \frac{\ln(1+2x) - 2x + 2x^2}{(2x)^3}$

a. 0
b. $\frac{8}{3}$
c. $\frac{4}{3}$
d. $\frac{1}{3}$
e. ∞

9. For what values of x does the series $\sum_{n=0}^{\infty} \frac{1}{(x-3)^n}$ converge?

a. $x > 4$ only
b. $x > 3$ only
c. $0 < x < 3$ only
d. $x < 1$ or $x > 3$ only
e. $x < 2$ or $x > 4$ only

10. In the Maclaurin series for $\frac{\sin(x^2)}{x}$ the coefficient of x^9 is

a. 1
b. $\frac{1}{3!}$
c. $\frac{1}{5!}$
d. $\frac{1}{7!}$
e. $\frac{1}{9!}$

11. (25 points) You are given: $\cos(x^2) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{4n}}{(2n)!} = 1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \cdots.$

a. (5 pt) If $f(x) = \cos(x^2)$, find $f^{(6)}(0)$.

b. (5 pt) If $f(x) = \cos(x^2)$, find $f^{(16)}(0)$.

c. (10 pt) Use the **quartic** (degree 4) Taylor polynomial approximation about $x = 0$ for $\cos(x^2)$ to estimate $\int_0^{0.1} \cos(x^2)\, dx$.

d. (5 pt) Your result in (c) is equal to $\int_0^{0.1} \cos(x^2)\, dx$ to within ± how much? Why?

12. (15 points) Find the interval of convergence for the series $\sum_{n=0}^{\infty} \frac{(x-6)^n}{2^n \sqrt{n}}$.

Be sure to identify each of the following and give reasons:

(1 pt) Center of Convergence: $a =$ ______________

Radius of Convergence: $R =$ ______________ (5 pt)

(1 pt) Right Endpoint: $x =$ ______________

At the Right Endpoint the Series $\left\{ \begin{array}{c} \text{Converges} \\ \text{Diverges} \end{array} \right\}$ (circle one) (3 pt)

(1 pt) Left Endpoint: $x =$ ______________

At the Left Endpoint the Series $\left\{ \begin{array}{c} \text{Converges} \\ \text{Diverges} \end{array} \right\}$ (circle one) (3 pt)

(1 pt) Interval of Convergence: ____________________________