Name________________________

1-11	/44	14	/10
12	/15	15	/20
13	/15	Total	/104

MATH 221 Final Fall 2009

Section 503 P. Yasskin

Multiple Choice: (4 points each. No part credit.)

1. Find the point where the line $x = 2t, \quad y = 4 - t, \quad z = -2 + t$ intersects the plane $x - y + z = -2$. At this point $x + y + z =$

a. -2

b. 0

c. 1

d. 4

e. 6

2. Find the plane tangent to the graph of $z = x^2y^3$ at the point $(2,1)$. The z-intercept is

a. -24

b. -16

c. -4

d. 0

e. 4

3. Find the line perpendicular to the cone $z^2 - x^2 - y^2 = 0$ at the point $P = (4,3,5)$. This line intersects the xy-plane at

a. $(3,4,0)$

b. $(-4,-3,0)$

c. $(8,6,0)$

d. $(-6,-8,0)$

e. $\left(\frac{4}{5}, \frac{3}{5}, 0\right)$

4. A box currently has length $L = 20$ cm which is increasing at 4 cm/sec, width $W = 15$ cm which is decreasing at 2 cm/sec, and height $H = 12$ cm which is increasing at 1 cm/sec. At what rate is the volume changing?

a. 3 cm^3/sec

b. 7 cm^3/sec

c. 540 cm^3/sec

d. 1500 cm^3/sec

e. 3600 cm^3/sec

5. Duke Skywater is flying the Millenium Eagle through a galactic dust storm. Currently, his position is $P = (30, -20, 10)$ and his velocity is $\vec{v} = (-4, 3, 12)$. He measures that currently the dust density is $\rho = 450$ and its gradient is $\vec{\nabla}\rho = (2, -2, 1)$. Find the current rate of change of the dust density as seen by Duke.

a. -2

b. 2

c. 110

d. 448

e. 560

6. Under the same conditions as in #5, in what **unit** vector direction should Duke travel to **decrease** the dust density as quickly as possible?

a. $(-2, 2, -1)$

b. $(2, -2, 1)$

c. $\left(\frac{4}{13}, \frac{-3}{13}, \frac{-12}{13}\right)$

d. $\left(\frac{-4}{13}, \frac{3}{13}, \frac{12}{13}\right)$

e. $\left(\frac{-2}{3}, \frac{2}{3}, \frac{-1}{3}\right)$

7. The point $(2,-1)$ is a critical point of the function $f = \dfrac{x^2y^2 + 2x - 4y}{xy}$. Use the Second Derivative Test to classify the point.

- **a**. Local Minimum
- **b**. Local Maximum
- **c**. Inflection Point
- **d**. Saddle Point
- **e**. Test Fails

8. Compute $\oint \vec{F} \cdot d\vec{s}$ counterclockwise around the rectangle $0 \le x \le \pi$ and $0 \le y \le 2\pi$ for $\vec{F} = (y\sin x + \sin y, x\cos y + \cos x)$.

HINT: Use the Fundamental Theorem of Calculus for Curves or Green's Theorem.

- **a**. -16π
- **b**. -8π
- **c**. 0
- **d**. 8π
- **e**. 16π

9. Compute $\iint \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ over the sphere $x^2 + y^2 + z^2 = 4$ with outward normal for $\vec{F} = (xy^2z, yz^2x, zx^2y)$.

HINT: Use Stokes' Theorem or Gauss' Theorem.

- **a**. 4π
- **b**. 12π
- **c**. $\dfrac{32}{3}\pi$
- **d**. $\dfrac{64}{3}\pi$
- **e**. 0

10. Find the area of the piece of the paraboloid $z = x^2 + y^2$ above the circle $x^2 + y^2 \le 2$.

Note: The paraboloid may be parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$.

a. $-\frac{\pi}{32}\left[\ln\left(2\sqrt{2}+3\right) - 102\sqrt{2}\right]$

b. $\frac{\pi}{2}\left[\ln\left(2\sqrt{2}+3\right) + 6\sqrt{2}\right]$

c. $\frac{13\pi}{3}$

d. $\frac{\pi}{6}(5^{3/2} - 1)$

e. $\frac{\pi}{6}(17^{3/2} - 1)$

11. Use Gauss' Theorem to compute $\iint \vec{F} \cdot d\vec{S}$ outward through the complete surface of the tetrahedron with vertices $(0,0,0)$, $(2,0,0)$, $(0,3,0)$ and $(0,0,6)$ for $\vec{F} = (x^2, xy, xz)$.

Note: The top of the tetrahedron is the plane $z = 6 - 3x - 2y$.

a. 4

b. 6

c. 9

d. 12

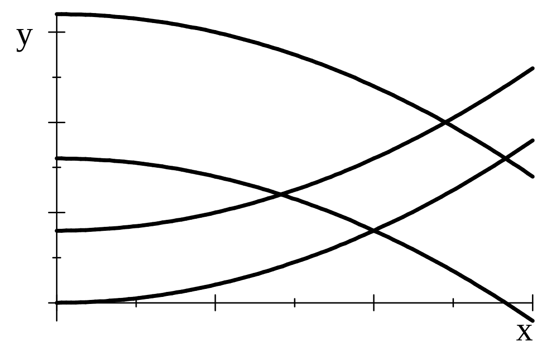
e. 16

Work Out: (Points indicated. Part credit possible. Show all work.)

12. (15 points) Compute $\iint_D x\,dx\,dy$ over the "diamond shaped" region D in the first quadrant bounded by the parabolas

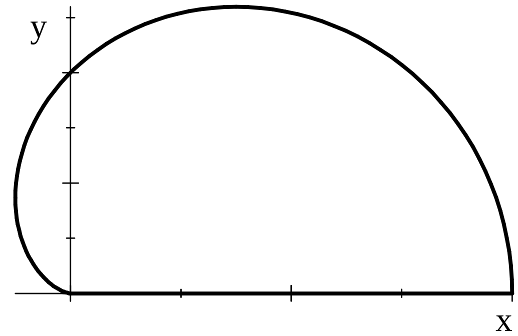
$y = 16 - x^2 \qquad y = 8 - x^2$

$y = 4 + x^2 \qquad$ and $\qquad y = x^2$



HINTS: Use the coordinates: $u = y + x^2$, $v = y - x^2$. Solve for x and y.

13. (15 points) Find the area and y-component of the centroid (center of mass with $\rho = 1$) of the upper half of the cardioid $r = 1 + \cos\theta$.



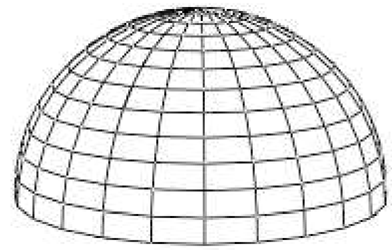
14. (10 points) Find 3 positive numbers x, y and z, whose sum is 120 such that $f(x,y,z) = xy^2z^3$ is a maximum.

15. (20 points) Verify Stokes' Theorem $\iint_H \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial H} \vec{F} \cdot d\vec{s}$

for the vector field $\vec{F} = (y, -x, xz + yz)$

and the hemisphere $z = \sqrt{9 - x^2 - y^2}$ oriented up.

Use the following steps:



a. Parametrize the boundary curve and compute the line integral:

$\vec{r}(\theta) =$

$\vec{v}(\theta) =$

$\vec{F}(\vec{r}(\theta)) =$

$\oint_{\partial H} \vec{F} \cdot d\vec{s} =$

b. Compute the surface integral using the parametrization:

$\vec{R}(\varphi, \theta) = (\quad 3\sin\varphi\cos\theta, \quad 3\sin\varphi\sin\theta, \quad 3\cos\varphi \quad)$

$\vec{e}_\varphi =$

$\vec{e}_\theta =$

$\vec{N} =$

$\vec{\nabla} \times \vec{F} =$

$\iint_H \vec{\nabla} \times \vec{F} \cdot d\vec{S} =$