

Name _____

MATH 251/221

Paper Homework 1

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Sections 200,511,500

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Prove the Pythagorean Identity for Vectors:

$$(\vec{u} \cdot \vec{v})^2 + |\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2$$

as follows: Consider the vectors $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$.

Compute each of the following by hand on paper. Show your work. Simplify where possible.

1. Write the algebraic definition for $\vec{u} \cdot \vec{v}$.

$$\vec{u} \cdot \vec{v} =$$

2. Write out $(\vec{u} \cdot \vec{v})^2$ to get 6 terms.

$$(\vec{u} \cdot \vec{v})^2 =$$

3. Write the algebraic definition for $\vec{u} \times \vec{v}$.

$$\vec{u} \times \vec{v} =$$

4. Write out $|\vec{u} \times \vec{v}|^2$ to get 9 terms.

$$|\vec{u} \times \vec{v}|^2 =$$

5. Add $(\vec{u} \cdot \vec{v})^2 + |\vec{u} \times \vec{v}|^2$ and cancel some terms.

$$(\vec{u} \cdot \vec{v})^2 + |\vec{u} \times \vec{v}|^2 =$$

6. Multiply out $|\vec{u}|^2 |\vec{v}|^2$.

$$|\vec{u}|^2 |\vec{v}|^2 =$$

7. Are the answers to (5) and (6) equal?