1. Compute the integral \( \iint x \, dA \) over the region in the first quadrant bounded by
\( y = 1 + x^2, \ y = 2 + x^2, \ y = 3 - x^2, \) and \( y = 5 - x^2. \)

a. Define the curvilinear coordinates \( u \) and \( v \) by \( y = u + x^2 \) and \( y = v - x^2. \)
What are the 4 boundaries in terms of \( u \) and \( v \)?

b. Solve for \( x \) and \( y \) in terms of \( u \) and \( v \). Express the results as a position vector.

\[
\vec{r}(u, v) = (x(u, v), y(u, v)) =
\]

c. Find the coordinate tangent vectors:
\[
\vec{e}_u = \frac{\partial \vec{r}}{\partial u} =
\]
\[
\vec{e}_v = \frac{\partial \vec{r}}{\partial v} =
\]

d. Compute the Jacobian determinant:
\[
\frac{\partial (x, y)}{\partial (u, v)} =
\]

e. Compute the Jacobian factor:
\[
J = \left| \frac{\partial (x, y)}{\partial (u, v)} \right| =
\]

f. Compute the integral:
\[
\iint x \, dA =
\]
2. Find the Jacobian for spherical coordinates. The position vector is given by

\[ \vec{R}(\rho, \theta, \varphi) = (\rho \sin \varphi \cos \theta, \rho \sin \varphi \sin \theta, \rho \cos \varphi) \]

a. Find the coordinate tangent vectors:

\[ \vec{e}_\rho = \frac{\partial \vec{R}}{\partial \rho} = \]

\[ \vec{e}_\theta = \frac{\partial \vec{R}}{\partial \theta} = \]

\[ \vec{e}_\varphi = \frac{\partial \vec{R}}{\partial \varphi} = \]

b. Compute the Jacobian determinant:

\[ \frac{\partial (x, y, z)}{\partial (\rho, \theta, \varphi)} = \]

c. Compute the Jacobian factor:

\[ J = \left| \frac{\partial (x, y, z)}{\partial (\rho, \theta, \varphi)} \right| = \]