1-8	/48
9	/15
10	/20
11	/10
12	/10
Total	/103

Name____________________________ ID______________________

MATH 251 Exam 2 Fall 2005

Sections 503 P. Yasskin

Multiple Choice: (6 points each. No part credit.)

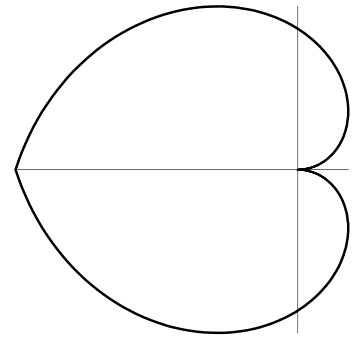
1. Find the volume of the solid under $z = 2x^2y$ above the region in the xy-plane between $y = x$ and $y = x^2$.

a. $\frac{2}{35}$
b. $\frac{35}{12}$
c. $\frac{12}{35}$
d. $\frac{1}{35}$
e. $\frac{1}{12}$

2. Compute $\iint \sin(x^2)\,dx\,dy$ over the triangle with vertices $(0,0)$, $(\sqrt{\pi},0)$, $(\sqrt{\pi},\sqrt{\pi})$.

a. $-\pi$
b. $-\sqrt{\pi}$
c. 1
d. $\sqrt{\pi}$
e. π

3. Find the area of the heart shaped region inside the polar curve $r = |\theta|$.



a. $\frac{\pi^3}{6}$
b. $\frac{\pi^3}{3}$
c. $\frac{4\pi^3}{3}$
d. $\frac{8\pi^3}{3}$
e. $\frac{16\pi^3}{3}$

4. Compute $\iiint \nabla \cdot \vec{F} \, dV$ on the solid cylinder bounded by

$x^2 + y^2 = 9$, $z = 0$ and $z = 5$ for the vector field $\vec{F} = (x^3, y^3, z(x^2 + y^2))$.

a. 45π
b. 90π
c. 360π
d. 810π
e. 900π

5. The solid hemisphere $0 \le z \le \sqrt{4 - x^2 - y^2}$ has density $\delta = z$. Find the total mass.

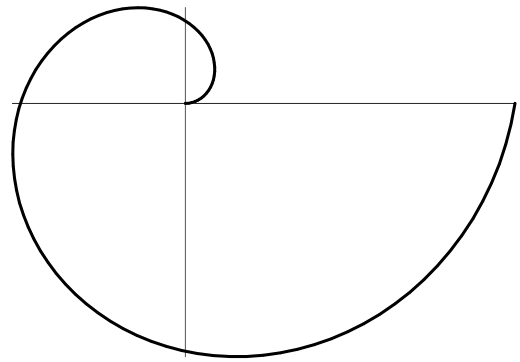
- **a.** $\pi/2$
- **b.** π
- **c.** 2π
- **d.** 4π
- **e.** 8π

6. The solid hemisphere $0 \le z \le \sqrt{4 - x^2 - y^2}$ has density $\delta = z$.

Find the z-component of the center of mass.

- **a.** 1
- **b.** $\frac{32}{15}\pi$
- **c.** $\frac{64}{15}\pi$
- **d.** $\frac{8}{15}$
- **e.** $\frac{16}{15}$

7. Compute $\int \sqrt{1+x^2+y^2}\,ds$ along the spiral $\vec{r}(t) = (t\cos t, t\sin t)$ from $(0,0)$ to $(2\pi, 0)$.



a. $\pi + \dfrac{\pi^3}{3}$

b. $2\pi + \dfrac{8\pi^3}{3}$

c. $\dfrac{(1+4\pi^2)^{3/2}}{3}$

d. $\dfrac{2(1+4\pi^2)^{3/2}}{3}$

e. $\dfrac{(1+4\pi^2)^{3/2}-1}{3}$

8. Compute $\int y\,dx - x\,dy$ along the spiral $\vec{r}(t) = (t\cos t, t\sin t)$ from $(0,0)$ to $(2\pi, 0)$.

a. $\dfrac{-8\pi^3}{3}$

b. $\dfrac{-4\pi^3}{3}$

c. $2\pi + \dfrac{8\pi^3}{3}$

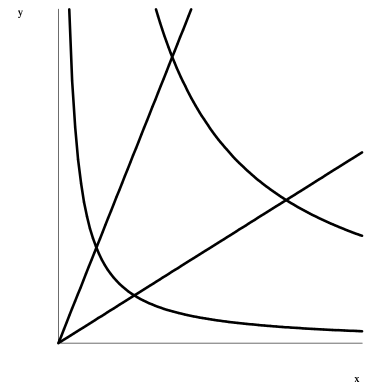
d. $\dfrac{4\pi^3}{3}$

e. $\dfrac{8\pi^3}{3}$

Work Out: (Part credit possible. Show all work.)

9. (15 points) Compute $\iint_R y^2\, dx\,dy$ over the diamond shaped region R bounded by

$$y = \frac{1}{x}, \qquad y = \frac{9}{x}, \qquad y = x, \qquad y = 4x$$

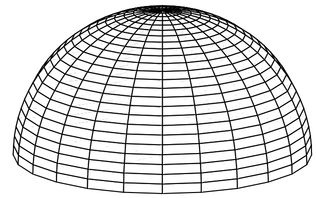


FULL CREDIT for integrating in the curvilinear coordinates (u,v) where $u^2 = xy$ and $v^2 = \frac{y}{x}$. (Solve for x and y.)

HALF CREDIT for integrating in rectangular coordinates.

10. (20 points) Consider the hemispherical surface

$$z = \sqrt{4 - x^2 - y^2}$$



which may be parametrized by

$$\vec{R}(\varphi, \theta) = (2\sin\varphi\cos\theta, 2\sin\varphi\sin\theta, 2\cos\varphi).$$

Find each of the following:

a. (2 pts) $\vec{e}_\varphi =$

b. (2 pts) $\vec{e}_\theta =$

c. (3 pts) $\vec{N} =$

d. (2 pts) $\left|\vec{N}\right| =$

e. (5 pts) The total mass of the surface if the surface density is $\delta = z$.

f. (6 pts) The z-component of the center of mass of the surface if the surface density is $\delta = z$.

11. (10 points) Consider the vector field $\vec{F} = (-y^3, x^3, z(x^2 + y^2))$ on the hemispherical surface of problem 10. Find each of the following:

a. (3 pts) $\nabla \times \vec{F} =$

b. (2 pts) $\nabla \times \vec{F}\left(\vec{R}(\varphi, \theta)\right) =$

c. (5 pts) $\iint \nabla \times \vec{F} \cdot d\vec{S}$ with normal pointing up.

12. (10 points) A bowl has the shape $z = \dfrac{x^2 + y^2}{3}$ for $z \leq 3$.

The bowl is filled with liquid of density $\rho = 12 - 2z$.

Find the total mass of the liquid.