Name________________________ ID__________________

MATH 251 Final Exam Fall 2005

Sections 503 P. Yasskin

1-12	/60
13	/20
14	/20
15	/10
Total	/110

Multiple Choice: (5 points each. No part credit.)

1. Find the area of the triangle with vertices $(1,1,1)$, $(2,2,4)$ and $(1,3,9)$.

a. $\sqrt{2}$

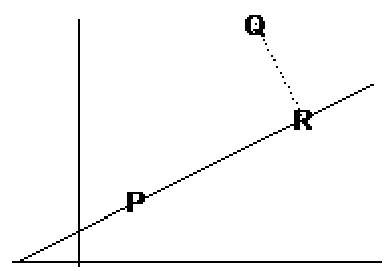
b. $3\sqrt{2}$

c. $6\sqrt{2}$

d. 4

e. 2

2. If you drop a perpendicular from the point $Q=(3,4)$ to the line through $P=(1,1)$ with direction $\vec{v}=(2,1)$, then the foot of the perpendicular is $R=$



a. $(3.2,2.0)$

b. $(3.4,2.2)$

c. $(3.6,2.3)$

d. $(3.8,2.4)$

e. $(4.0,2.5)$

3. Find the point where the line $(x,y,z) = (3,2,1) + t(1,2,3)$ intersects the plane $x - y + z = -2$.
At this point, $x + y + z =$

a. -6

b. -2

c. 0

d. 2

e. 6

4. Use the linear approximation to $\ln(x) + 2\ln(y)$ at $(x,y) = (1,1)$ to estimate $\ln(1.3) + 2\ln(.9)$.

a. 1.1

b. 1.05

c. 0.5

d. 0.1

e. 0.05

5. Find the equation of the plane tangent to the hyperboloid $z^2 - x^2 - y^2 = 4$ at the point $(1,2,3)$.

a. $-2x + 4y + 6z = 24$

b. $x + 2y + 3z = 14$

c. $2x + 4y - 6z = -8$

d. $(x,y,z) = (1 - 2t, 2 - 4t, 3 + 6t)$

e. $(x,y,z) = (1 - 2t, 2 + 4t, 3 + 6t)$

6. Duke Skywater is travelling through the galaxy. He is currently at the point with galactic coordinates $(40, 25, 53)$ (in lightyears), and his velocity is $(.2, -.1, .3)$ (in lightyears/year). He measures the polaron density to be $U = 4300$ polarons/cm^3 and its gradient to be $\vec{\nabla} U = (3, 2, 1)$ polarons/cm^3/lightyear. Find the rate at which he sees the polaron density changing.

a. 223

b. 21.4

c. 0.11

d. -0.11

e. 0.7

7. The function $f(x,y) = \frac{1}{4}x^3 - xy + \frac{2}{27}y^3$ has a critical point at $(x,y) = (2,3)$ which, according to the Second Derivative Test, is

a. a local minimum.

b. a local maximum.

c. a saddle point.

d. an inflection point.

e. The Second Derivative Test FAILS.

8. Rewrite the polar equation $r^2 = \sin 2\theta$ in rectangular coordinates.

a. $x^4 + y^4 = 2xy$

b. $(x^2 + y^2)^2 = 2xy$

c. $(x^2 + y^2)^{3/2} = 2y$

d. $(x^2 + y^2)^{3/2} = 2x$

e. $x^3 + y^3 = 2y$

9. For an ideal gas, the pressure, P, is a function of the temperature, T, and volume, V, given by $P = \frac{kT}{V}$ where k is a constant. For a certain sample of gas the current values are

$$T = 250°\text{K} \qquad V = 5\ \text{m}^3 \qquad k = 2\ \frac{\text{kPa} \cdot \text{m}^3}{°\text{K}} \qquad \text{and consequently} \qquad P = 100\ \text{kPa}$$

If the volume and temperature are increasing at

$$\frac{dV}{dt} = 0.2\ \frac{\text{m}^3}{\text{sec}} \qquad \text{and} \qquad \frac{dT}{dt} = 6\ \frac{°\text{K}}{\text{sec}}$$

is the pressure increasing or decreasing and at what rate?

a. decreasing at $1.6\ \frac{\text{kPa}}{\text{sec}}$

b. decreasing at $8\ \frac{\text{kPa}}{\text{sec}}$

c. increasing at $1.6\ \frac{\text{kPa}}{\text{sec}}$

d. increasing at $8\ \frac{\text{kPa}}{\text{sec}}$

e. Pressure is constant.

10. A particle moves along the curve $\vec{r}(t) = (t, t^2, t^3)$ from $(1,1,1)$ to $(2,4,8)$ due to the force $\vec{F} = (z, y, x)$. Find the work done by the force.

a. $\frac{70}{3}$

b. 24

c. $\frac{45}{2}$

d. $\frac{96}{5}$

e. $\frac{93}{5}$

11. Compute $\int_{(1,1,1)}^{(1/e,\,1/e,\,e^2)} \vec{F} \cdot d\vec{s}$ where $\vec{F} = (z, z, x + y)$ along the curve $\vec{r}(t) = (e^{-t}, e^{-t}, e^{2t})$.

HINT: Note $\vec{F} = \vec{\nabla} f$ where $f = xz + yz$.

a. $4 - 4e$

b. $4e - 4$

c. $4 - 2e$

d. $2 - 2e$

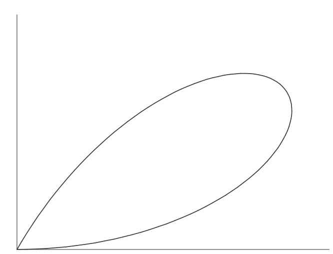
e. $2e - 2$

12. Use Green's Theorem to compute

$$\oint (y^2 + 2y - \ln x)\, dx + (4x + 2xy + e^y)\, dy$$

counterclockwise around the polar curve

$$r = \sin(3\theta) \quad \text{for} \quad 0 \le \theta \le \frac{\pi}{3}.$$



a. $\frac{\pi}{2}$

b. $\frac{\pi}{3}$

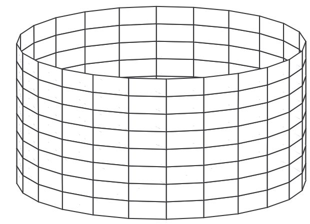
c. $\frac{\pi}{4}$

d. $\frac{\pi}{6}$

e. $\frac{\pi}{12}$

Work Out: (Points indicated. Part credit possible.)

13. (20 points) Verify Stokes' Theorem $\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial C} \vec{F} \cdot d\vec{s}$



for the vector field $\vec{F} = (-yz, xz, z^2)$ and

the cylinder $x^2 + y^2 = 4$ for $1 \le z \le 3$ oriented out.

Be sure to check and explain the orientations.

Use the following steps:

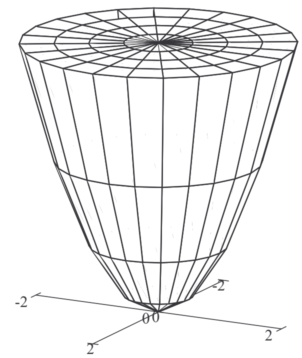
a. The cylindrical surface may be parametrized by $\vec{R}(\theta, z) = (2\cos\theta, 2\sin\theta, z)$.
Compute the surface integral:
Successively find: $\vec{e}_\theta$, $\vec{e}_z$, $\vec{N}$, $\vec{\nabla} \times \vec{F}$, $\vec{\nabla} \times \vec{F}\big(\vec{R}(\theta, z)\big)$, $\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S}$

b. Let U be the upper circle. Parametrize U and compute the line integral.
Successively find: $\vec{r}(\theta)$, $\vec{v}(\theta)$, $\vec{F}(\vec{r}(\theta))$, $\oint_U \vec{F} \cdot d\vec{s}$.

c. Let L be the lower circle. Parametrize L and compute the line integral.
Successively find: $\vec{r}(\theta)$, $\vec{v}(\theta)$, $\vec{F}(\vec{r}(\theta))$, $\oint_L \vec{F} \cdot d\vec{s}$.

d. Combine $\oint_U \vec{F} \cdot d\vec{s}$ and $\oint_L \vec{F} \cdot d\vec{s}$ to get $\oint_{\partial C} \vec{F} \cdot d\vec{s}$.

14. (20 points) Verify Gauss' Theorem $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV = \iint\limits_{\partial V} \vec{F} \cdot d\vec{S}$



for the vector field $\vec{F} = (xz, yz, z^2)$ and

the volume above the paraboloid P: $z = x^2 + y^2$ for $z \le 4$

and below the disk D: $x^2 + y^2 \le 4$ with $z = 4$.

Be sure to check and explain the orientations.

Use the following steps.

a. Compute the divergence $\vec{\nabla} \cdot \vec{F}$ and the volume integral $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV$.

b. Parametrize the disk, D, and compute the surface integral:
Successively find: $\vec{R}(r,\theta)$, $\vec{e}_r$, $\vec{e}_\theta$, $\vec{N}$, $\vec{F}\left(\vec{R}(r,\theta)\right)$, $\iint\limits_D \vec{F} \cdot d\vec{S}$.

c. The paraboloid, P, may be parametrize as $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$.
Compute the surface integral:
Successively find: $\vec{e}_r$, $\vec{e}_\theta$, $\vec{N}$, $\vec{F}\big(\vec{R}(r,\theta)\big)$, $\iint\limits_P \vec{F} \cdot d\vec{S}$.

d. Combine $\iint\limits_D \vec{F} \cdot d\vec{S}$ and $\iint\limits_P \vec{F} \cdot d\vec{S}$ to get $\iint\limits_{\partial V} \vec{F} \cdot d\vec{S}$.

15. $(10$ points$)$ Find the point on the plane $z = 2 - 2x - y$ that is closest to the origin.