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MATH 251 Quiz 1 Fall 2005

Sections 503 Solutions P. Yasskin

Multiple Choice & Work Out: (5 points each)

**1.** A triangle has vertices $A = (0,3,2)$, $B = (-2,3,0)$ and $C = (-2,0,3)$. Find the angle at vertex $B$.

**a.** $\frac{\pi}{6}$

**b.** $\frac{\pi}{3}$ Correct Choice

**c.** $\frac{\pi}{2}$

**d.** $\frac{2\pi}{3}$

**e.** $\frac{5\pi}{6}$

$$\overrightarrow{BA} = A - B = (2,0,2) \qquad \overrightarrow{BC} = C - B = (0,-3,3) \qquad \overrightarrow{BA} \cdot \overrightarrow{BC} = 6$$

$$\left|\overrightarrow{BA}\right| = \sqrt{4+4} = 2\sqrt{2} \qquad \left|\overrightarrow{BC}\right| = \sqrt{9+9} = 3\sqrt{2}$$

$$\cos\theta = \frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{\left|\overrightarrow{BA}\right|\left|\overrightarrow{BC}\right|} = \frac{6}{2\sqrt{2}\,3\sqrt{2}} = \frac{1}{2} \qquad \Rightarrow \qquad \theta = 60^\circ = \frac{\pi}{3}$$

**2.** A triangle has vertices $A = (0,3,2)$, $B = (-2,3,0)$ and $C = (-2,0,3)$. Find the area of the triangle.

**a.** $15$

**b.** $30$

**c.** $2\sqrt{3}$

**d.** $3\sqrt{3}$ Correct Choice

**e.** $6\sqrt{3}$

$$\overrightarrow{BA} = A - B = (2,0,2) \qquad \overrightarrow{BC} = C - B = (0,-3,3)$$

$$\overrightarrow{BA} \times \overrightarrow{BC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 0 & 2 \\ 0 & -3 & 3 \end{vmatrix} = \hat{\imath}(6) - \hat{\jmath}(6) + \hat{k}(-6) = (6,-6,-6)$$

$$\text{Area} = \frac{1}{2}\left|\overrightarrow{BA} \times \overrightarrow{BC}\right| = \frac{1}{2}\sqrt{36+36+36} = 3\sqrt{3}$$

**3.** If $\vec{u}$ points Up (away from the center of the earth) and $\vec{v}$ points NorthEast, then $\vec{u} \times \vec{v}$ points

**a.** Up

**b.** Down

**c.** SouthEast

**d.** SouthWest

**e.** NorthWest   Correct Choice

Put your fingers Up with the palm facing NorthEast, your thumb points NorthWest.

**4.** Find the equation of the plane which is perpendicular to the line $(x,y,z) = (2-3t, 3+t, 1-t)$ and passes through the point $(-1,4,3)$.

**a.** $2x+3y+z = 13$

**b.** $2x+3y+z = -4$

**c.** $-3x+y-z = 4$   Correct Choice

**d.** $-3x+y-z = -4$

**e.** $-x+4y+3z = 13$

The normal to the plane is the tangent vector to the line: $\vec{N} = \vec{v} = (-3,1,-1)$.
A point on the plane is $P = (-1,4,3)$  So the plane is $\vec{N} \cdot X = \vec{N} \cdot P$ or
$-3x+y-z = 3+4-3 = 4$

**5.** Consider the set of all points $P$ such that the distance from $P$ to $(3,3,3)$ is twice the distance from $P$ to $(0,0,0)$. This set of points is a sphere. Find its center and radius.

Let $P = (x,y,z)$, $O = (0,0,0)$ and $Q = (3,3,3)$. Then $\left|\overrightarrow{PO}\right| = \sqrt{x^2+y^2+z^2}$ and $\left|\overrightarrow{PQ}\right| = \sqrt{(x-3)^2+(y-3)^2+(z-3)^2}$. The definition of $P$ is $\left|\overrightarrow{PQ}\right| = 2\left|\overrightarrow{PO}\right|$. So

$$\sqrt{(x-3)^2+(y-3)^2+(z-3)^2} = 2\sqrt{x^2+y^2+z^2}$$
$$(x-3)^2+(y-3)^2+(z-3)^2 = 4(x^2+y^2+z^2)$$
$$x^2+y^2+z^2-6x-6y-6z+27 = 4x^2+4y^2+4z^2$$
$$27 = 3x^2+3y^2+3z^2+6x+6y+6z$$
$$x^2+y^2+z^2+2x+2y+2z = 9$$
$$x^2+y^2+z^2+2x+2y+2z+3 = 12$$
$$(x+1)^2+(y+1)^2+(z+1)^2 = 12$$