Name________________________ ID_____________

MATH 251 Quiz 3 Fall 2005

Sections 503 Solutions P. Yasskin

1-4	/20
5	/ 5
Total	/25

Multiple Choice & Work Out: (5 points each)

1. For the function $f(x,y) = x\cos(xy)$ which partial derivative is incorrect?

a. $\dfrac{\partial f}{\partial x} = \cos(xy) - xy\sin(xy)$

b. $\dfrac{\partial f}{\partial y} = -x^2\sin(xy)$

c. $\dfrac{\partial^2 f}{\partial x^2} = -y\sin(xy) - x^2 y\cos(xy)$ Correct Choice

d. $\dfrac{\partial^2 f}{\partial x \partial y} = -2x\sin(xy) - x^2 y\cos(xy)$

e. $\dfrac{\partial^2 f}{\partial y \partial x} = -2x\sin(xy) - x^2 y\cos(xy)$

Use chain rule and product rule: $\dfrac{\partial^2 f}{\partial x^2} = -2y\sin(xy) - xy^2\cos(xy)$

2. Find the equation of the plane tangent to $z = x^2y^3$ at the point $(2,1,4)$.

a. $z = -4x - 12y + 24$

b. $z = -4x - 12y + 4$

c. $z = 4x + 12y + 4$

d. $z = 4x + 12y - 8$

e. $z = 4x + 12y - 16$ Correct Choice

$f(x,y) = x^2y^3 \qquad f(2,1) = 4$

$\dfrac{\partial f}{\partial x} = 2xy^3 \qquad \dfrac{\partial f}{\partial x}(2,1) = 4$

$\dfrac{\partial f}{\partial y} = 3x^2y^2 \qquad \dfrac{\partial f}{\partial y}(2,1) = 12$

$$z = f_{\tan}(x,y) = f(2,1) + \frac{\partial f}{\partial x}(2,1)(x-2) + \frac{\partial f}{\partial y}(2,1)(y-1) = 4 + 4(x-2) + 12(y-1)$$

$z = 4x + 12y - 16$

3. The plane tangent to $z = f(x,y) = xy^2 - x^2$ at the point $(1,2,-3)$ is
$z = f_{\tan}(x,y) = 3 + 2(x-1) + 4(y-2)$. Use this information to approximate $f(1.1, 1.8)$.

a. 2

b. 2.4 Correct Choice

c. 3.6

d. 4

e. 12.4

$$f(1.1,1.8) \approx f_{\tan}(1.1,1.8) = 3 + 2(1.1-1) + 4(1.8-2) = 3 + 2(.1) + 4(-.2) = 2.4$$

4. Consider a function $g(x,y)$. If $g(2,3) = 4$, $\dfrac{\partial g}{\partial x} = 5$, and $\dfrac{\partial g}{\partial y} = 1$, estimate $g(1.9, 3.3)$.

a. 3.8 Correct Choice

b. 4.2

c. 4.8

d. 10

e. 16.8

$$g_{\tan}(x,y) = g(2,3) + \frac{\partial g}{\partial x}(2,3)(x-2) + \frac{\partial g}{\partial y}(2,3)(y-3) = 4 + 5(x-2) + 1(y-3)$$

$$g(1.9,3.3) \approx g_{\tan}(1.9,3.3) = 4 + 5(1.9-2) + 1(3.3-3) = 4 + 5(-.1) + 1(.3) = 3.8$$

5. The mass of a body is $M = \rho V$ where ρ is its density and V is its volume.
If the density is measured to be $\rho = 1.2\dfrac{\text{g}}{\text{cm}^3}$ with an uncertainty of $\Delta\rho = \pm 0.01\dfrac{\text{g}}{\text{cm}^3}$
and the volume is measured to be $V = 2\,\text{cm}^3$ with an uncertainty of $\Delta V = \pm 0.02\,\text{cm}^3$,
then the mass is $M = 2.4\,\text{g}$. Use differentials to estimate the uncertainty in the mass.
NOTE: The uncertainty in a quantity is its differential.

$$\frac{\partial M}{\partial \rho} = V = 2 \qquad \frac{\partial M}{\partial V} = \rho = 1.2$$

$$\Delta M \approx dM = \frac{\partial M}{\partial \rho}d\rho + \frac{\partial M}{\partial V}dV = 2(\pm 0.01) + 1.2(\pm 0.02) = \pm 0.044\,\text{g}$$