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1-3	/15
4	/ 5
5	/10
Total	/30

MATH 251 Quiz 5 Fall 2005

Sections 503 Solutions P. Yasskin

Multiple Choice: (5 points each)

1. Compute $\int_1^2 \int_1^x y \, dy \, dx$.

a. $-\frac{1}{3}$

b. $\frac{1}{3}$

c. $\frac{2}{3}$ Correct Choice

d. $\frac{7}{6}$

e. $\frac{4}{3}$

$$\int_1^2 \int_1^x y \, dy \, dx = \int_1^2 \left[\frac{y^2}{2} \right]_{y=1}^{x} dx = \int_1^2 \frac{x^2}{2} - \frac{1}{2} \, dx = \left[\frac{x^3}{6} - \frac{x}{2} \right]_1^2 = \left[\frac{8}{6} - \frac{2}{2} \right] - \left[\frac{1}{6} - \frac{1}{2} \right] = \frac{2}{3}$$

2. Find the volume under the surface $z = 2x^2 y$ above the triangle with vertices $(0,0)$, $(1,2)$ and $(0,4)$.

a. $-\frac{1}{3}$

b. $\frac{1}{3}$

c. $\frac{2}{3}$

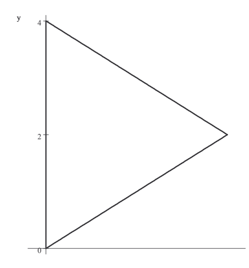
d. $\frac{7}{6}$

e. $\frac{4}{3}$ Correct Choice

$$\int_0^1 \int_{2x}^{4-2x} 2x^2 y \, dy \, dx = \int_0^1 \left[x^2 y^2 \right]_{y=2x}^{4-2x} dx = \int_0^1 x^2 \left[(4-2x)^2 - (2x)^2 \right] dx$$

$$= \int_0^1 x^2 (16 - 16x) \, dx = \int_0^1 16x^2 - 16x^3 \, dx = \left[\frac{16}{3} x^3 - 4x^4 \right]_0^1$$

$$= \left[\frac{16}{3} - 4 \right] = \frac{4}{3}$$



3. Compute $\iiint_D \sqrt{x^2 + y^2} \, dV$ over the region D bounded by the paraboloid $z = 9 - x^2 - y^2$ and the xy-plane.

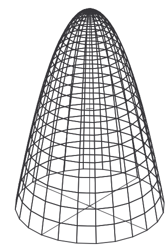
a. $\frac{4\pi}{5} 3^4$ Correct Choice

b. $\frac{\pi}{2} 3^4$

c. $\frac{\pi}{2} 3^5$

d. $2\pi 3^4$

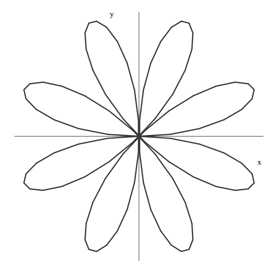
e. $2\pi 3^5$



$$\iiint_D \sqrt{x^2+y^2}\ dV = \int_0^{2\pi}\int_0^3\int_0^{9-r^2} r\ r\ dz\ dr\ d\theta = 2\pi\int_0^3\int_0^{9-r^2} r^2\ dz\ dr = 2\pi\int_0^3 \Big[r^2 z \ \Big]_{z=0}^{9-r^2} dr$$

$$= 2\pi\int_0^3 r^2(9-r^2)\ dr = 2\pi\Big[\frac{9r^3}{3} - \frac{r^5}{5}\Big]_0^3 = 2\pi\Big[\frac{9\cdot 3^3}{3} - \frac{3^5}{5}\Big] = 2\pi 3^4\Big[1-\frac{3}{5}\Big] = \frac{4\pi}{5}3^4$$

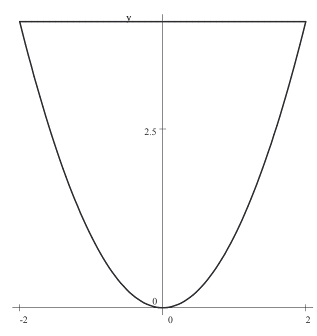
4. (5 points) Find the area enclosed by **ONE** loop of the daisy $r = \sin 4\theta$:



$$A = \iint 1\ dA = \int_0^{\pi/4}\int_0^{\sin 4\theta} r\ dr\ d\theta = \int_0^{\pi/4} \frac{r^2}{2}\Big|_{r=0}^{\sin 4\theta} d\theta = \int_0^{\pi/4} \frac{\sin^2 4\theta}{2}\ d\theta$$

$$= \int_0^{\pi/4} \frac{1-\cos 8\theta}{4}\ d\theta = \frac{1}{4}\Big[\theta - \frac{\sin 8\theta}{8}\Big]_{\theta=0}^{\pi/4} = \frac{1}{4}\frac{\pi}{4} = \frac{\pi}{16}$$

5. (10 points) Find the mass M and center of mass $(\bar{x},\bar{y})$ of the region above the parabola $y = x^2$ below the line $y = 4$, if the density is $\rho = y$.
(5 points for setup.)
HINT: By symmetry, $\bar{x} = 0$. So you only need to compute $\bar{y}$.



$$M = \iint \rho\ dA = \int_{-2}^{2}\int_{x^2}^{4} y\ dy\ dx = \int_{-2}^{2}\Big[\frac{y^2}{2}\Big]_{y=x^2}^{4} dx = \int_{-2}^{2} 8 - \frac{x^4}{2}\ dx = \Big[8x - \frac{x^5}{10}\Big]_{-2}^{2}$$

$$= 2\Big[16 - \frac{32}{10}\Big] = 32\Big[1-\frac{1}{5}\Big] = \frac{128}{5}$$

$$y\text{-mom} = \iint y\rho\ dA = \int_{-2}^{2}\int_{x^2}^{4} y^2\ dy\ dx = \int_{-2}^{2}\Big[\frac{y^3}{3}\Big]_{y=x^2}^{4} dx = \int_{-2}^{2}\frac{64}{3} - \frac{x^6}{3}\ dx$$

$$= \frac{1}{3}\Big[64x - \frac{x^7}{7}\Big]_{-2}^{2} = \frac{2}{3}\Big[128 - \frac{128}{7}\Big] = \frac{256}{3}\Big(\frac{6}{7}\Big) = \frac{512}{7}$$

$$\bar{y} = \frac{y\text{-mom}}{M} = \frac{512}{7}\frac{5}{128} = \frac{20}{7}$$