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1-12	/60
13	/20
14	/10
15	/15
Total	/105

MATH 251 Final Exam Spring 2006

Sections 506 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Let L be the line $\vec{r}(t) = (6+7t, -3-4t, 5-2t)$. Find the equation of the plane perpenducular to L that contains the point $(3,-5,2)$.

a. $3(x-7) - 5(y+4) + 2(z+2) = 0$

b. $3(x-7) + 5(y+4) + 2(z+2) = 0$

c. $7(x-3) - 4(y+5) - 2(z-2) = 0$

d. $7(x-3) + 4(y+5) - 2(z-2) = 0$

e. $6(x-3) - 3(y+5) + 5(z-2) = 0$

2. Find the equation of the plane tangent to the graph of $z = x\sin y$ at $(x,y) = \left(2, \frac{\pi}{3}\right)$.

a. $z = \frac{1}{2}x + \sqrt{3}\,y - \frac{\pi}{\sqrt{3}} + \sqrt{3} - 1$

b. $z = \frac{1}{2}x + \sqrt{3}\,y - \frac{\pi}{\sqrt{3}} + \sqrt{3}$

c. $z = \frac{1}{2}x + \sqrt{3}\,y + \sqrt{3} - 1$

d. $z = \frac{\sqrt{3}}{2}x + y - \frac{\pi}{3}$

e. $z = \frac{\sqrt{3}}{2}x + y + \sqrt{3}$

3. Find the equation of the line perpendicular to the surface $xy + z^2 = 6$ at the point $(1, 2, 2)$.

a. $x = 2 + t, y = -1 + 2t, z = 4 + 2t$

b. $x = 2 + t, y = -1 - 2t, z = 4 + 2t$

c. $x = 2 + t, y = 1 + 2t, z = 4 + 2t$

d. $x = 1 + 2t, y = 2 - t, z = 2 + 4t$

e. $x = 1 + 2t, y = 2 + t, z = 2 + 4t$

4. $\lim_{(x,y)\to(0,0)} \dfrac{x^2y^2 - 2y}{y - xy} =$

a. -2

b. 0

c. 1

d. 2

e. Does Not Exist

5. The radius and height of a cylinder are currently $r = 10$ cm and $h = 6$ cm. If the radius is increasing at $\dfrac{dr}{dt} = 2 \dfrac{\text{cm}}{\text{min}}$ and the volume is increasing at $\dfrac{dV}{dt} = 40\pi \dfrac{\text{cm}^3}{\text{min}}$, is the height increasing or decreasing and at what rate?

a. decreasing at $2 \dfrac{\text{cm}}{\text{min}}$

b. decreasing at $\dfrac{4}{5} \dfrac{\text{cm}}{\text{min}}$

c. increasing at $2 \dfrac{\text{cm}}{\text{min}}$

d. increasing at $\dfrac{4}{5} \dfrac{\text{cm}}{\text{min}}$

e. The height is constant.

6. Han Duet is flying the Millenium Eagle through a radion field with density $\rho = z(x+y)$. He is currently located at $(-4,3,5)$ in galactic coordinates. In what direction should he fly to decrease the radion density as fast as possible?

a. $(-5,5,1)$

b. $(-5,-5,1)$

c. $(5,5,-1)$

d. $(28,-21,35)$

e. $(-28,21,-35)$

7. Han Duet is flying the Millenium Eagle through a radion field with density $\rho = z(x+y)$. He is currently located at $(-4,3,5)$ in galactic coordinates and has velocity $\vec{v} = (0.2,-0.1,0.3)$. What does he see as the time rate of change of the radion density?

a. 0.2

b. -0.2

c. 1.2

d. -1.2

e. 0.4

8. Find the volume below $z = 2x^2y$ above the region in the xy-plane bounded by $y = 0$, $y = x^2$ and $x = 2$.

a. $\frac{32}{5}$

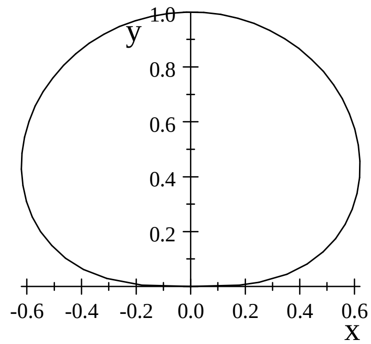
b. $\frac{32}{3}$

c. $\frac{128}{7}$

d. 32

e. $\frac{512}{9}$

9. The graph of the polar curve $r = \sqrt{\sin(\theta)}$ is shown at the right. Find the area enclosed.



a. 1.2

b. 1.0

c. 0.8

d. $\frac{\pi}{3}$

e. $\frac{\pi}{4}$

10. Compute $\int_{(-1,0,-1)}^{(2,0,8)} \vec{F} \cdot d\vec{s}$ where $\vec{F} = (3x^2, 2y, 1)$ along the curve

$\vec{r}(t) = (t\cos(\pi t), t^2\sin(\pi t), t^3\cos(\pi t))$.

HINT: Note $\vec{F} = \vec{\nabla} f$ where $f = x^3 + y^2 + z$.

a. 8

b. 9

c. 12

d. 15

e. 18

11. Compute $\oint (\ln x - 3xe^y)\,dx + (x^2 e^y)\,dy$ along the closed curve which travels along the straight line from $(0,0)$ to $(1,0)$, along the straight line from $(1,0)$ to $(1,1)$ and along $y = x^2$ from $(1,1)$ to $(0,0)$.

a. $5 - 5e$

b. $5e - 5$

c. $5 - \frac{5}{2}e$

d. $\frac{5}{2}e - 5$

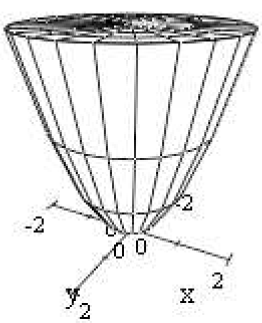
e. Diverges

12. Compute $\iint_{\partial P} \vec{F} \cdot d\vec{S}$ over the complete surface of the solid paraboloid

$$x^2 + y^2 \le z \le 4$$

with outward normal, for the vector field

$$\vec{F} = (x^3, y^3, x + y)$$



a. $\frac{16\pi}{3}$

b. $\frac{32\pi}{3}$

c. 16π

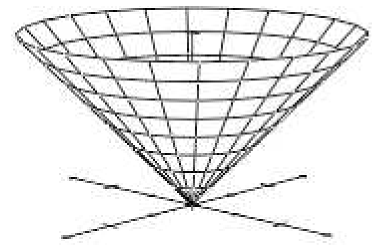
d. 32π

e. 48π

Original problem had $\vec{F} = (x^3, y^3, z)$ and no correct answer. Problem thrown out.

13. (20 points) Verify Stokes' Theorem

$$\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial C} \vec{F} \cdot d\vec{s}$$



for the vector field $\vec{F} = (-yz^2, xz^2, z^3)$ and the cone

$z = \sqrt{x^2 + y^2}$ for $z \le 3$ oriented down and out.

Be sure to check and explain the orientations.

Use the following steps:

a. The conical surface may be parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$.
Compute the surface integral:
Successively find: $\vec{e}_r$, $\vec{e}_\theta$, $\vec{N}$, $\vec{\nabla} \times \vec{F}$, $\vec{\nabla} \times \vec{F}\left(\vec{R}(r,\theta)\right)$, $\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S}$

Problem Continued

b. Parametrize the boundary circle ∂C and compute the line integral.
Successively find: $\vec{r}(\theta)$, $\vec{v}(\theta)$, $\vec{F}(\vec{r}(\theta))$, $\oint_{\partial C} \vec{F} \cdot d\vec{s}$.

14. $\big(10$ points$\big)$ Find all critical points of $f(x,y) = xy - \frac{1}{3}x^3 - y^2$ and classify each of them as either a local minimum, a local maximum or a a saddle point. Justify your answers.

15. (15 points) Find the mass and z-component of the center of mass of the solid hemisphere

$$0 \le x \le \sqrt{4 - y^2 - z^2}$$

if the density is given by $\delta = 3 + z$.