Name________________________ ID_______________

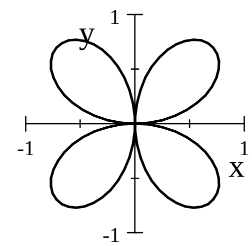
1-2	/20
3	/10
Total	/30

MATH 251 Quiz 5 Spring 2006

Sections 506 Solutions P. Yasskin

Multiple Choice: (10 points each)

1. Find the area enclosed by **ONE** loop of the daisy $r = \sin 2\theta$:



a. $\frac{\pi}{8}$ Correct Choice

b. $\frac{\pi}{4}$

c. $\frac{\pi}{2}$

d. π

e. 2π

$$A = \iint 1\,dA = \int_0^{\pi/2}\int_0^{\sin 2\theta} r\,dr\,d\theta = \int_0^{\pi/2} \frac{r^2}{2}\bigg|_{r=0}^{\sin 2\theta} d\theta = \int_0^{\pi/2} \frac{\sin^2 2\theta}{2}\,d\theta$$

$$= \int_0^{\pi/2} \frac{1-\cos 4\theta}{4}\,d\theta = \frac{1}{4}\left[\theta - \frac{\sin 4\theta}{4}\right]_{\theta=0}^{\pi/2} = \frac{1}{4}\frac{\pi}{2} = \frac{\pi}{8}$$

2. Compute $\iiint\limits_D \sqrt{x^2+y^2}\;dV$ over the region D bounded by the paraboloid $z = 9 - x^2 - y^2$ and the xy-plane.

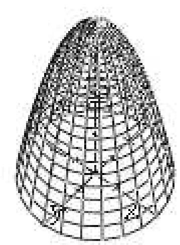
a. $2\pi 3^4$

b. $\frac{\pi}{2}3^4$

c. $\frac{\pi}{2}3^5$

d. $\frac{4\pi}{5}3^4$ Correct Choice

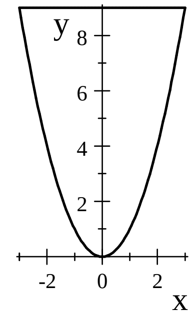
e. $2\pi 3^5$



$$\iiint\limits_D \sqrt{x^2+y^2}\;dV = \int_0^{2\pi}\int_0^3\int_0^{9-r^2} r\;r\;dz\;dr\;d\theta = 2\pi\int_0^3\int_0^{9-r^2} r^2\;dz\;dr = 2\pi\int_0^3\left[r^2 z\right]_{z=0}^{9-r^2} dr$$

$$= 2\pi\int_0^3 r^2(9-r^2)\;dr = 2\pi\left[\frac{9r^3}{3} - \frac{r^5}{5}\right]_0^3 = 2\pi\left[\frac{9\cdot 3^3}{3} - \frac{3^5}{5}\right] = 2\pi 3^4\left[1 - \frac{3}{5}\right] = \frac{4\pi}{5}3^4$$

3. (10 points) Find the mass M and center of mass $(\bar{x},\bar{y})$
of the region above the parabola $y = x^2$
below the line $y = 9$, if the density is $\rho = y$.
(5 points for setup.)
HINT: By symmetry, $\bar{x} = 0$. So you only need to compute $\bar{y}$.



$$M = \int\int \rho\, dA = \int_{-3}^{3}\int_{x^2}^{9} y\, dy\, dx = \int_{-3}^{3}\left[\frac{y^2}{2}\right]_{y=x^2}^{9} dx = \int_{-3}^{3} \frac{81}{2} - \frac{x^4}{2}\, dx = \left[\frac{81}{2}x - \frac{x^5}{10}\right]_{-3}^{3}$$

$$= 2\left(\frac{3^5}{2} - \frac{3^5}{10}\right) = 3^5\left(1 - \frac{1}{5}\right) = \frac{4 \cdot 3^5}{5}$$

$$M_x = \int\int y\rho\ dA = \int_{-3}^{3}\int_{x^2}^{9} y^2\, dy\, dx = \int_{-3}^{3}\left[\frac{y^3}{3}\right]_{y=x^2}^{9} dx = \frac{1}{3}\int_{-3}^{3} 9^3 - x^6\, dx$$

$$= \frac{1}{3}\left[9^3 x - \frac{x^7}{7}\right]_{-3}^{3} = \frac{2}{3}\left[3^7 - \frac{3^7}{7}\right] = 2 \cdot 3^6\left(\frac{6}{7}\right) = \frac{4 \cdot 3^7}{7}$$

$$\bar{y} = \frac{M_x}{M} = \frac{4 \cdot 3^7}{7}\,\frac{5}{4 \cdot 3^5} = \frac{45}{7}$$