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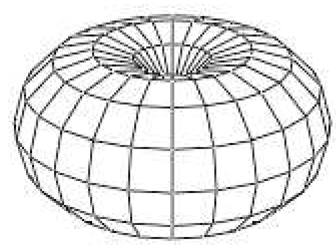
1	/ 5
2	/10
2	/10
Total	/30

MATH 251 Quiz 6 Spring 2006

Sections 506 Solutions P. Yasskin

Multiple Choice: (5 points each)

1. (5 points) Which of the following integrals will give the volume of the donut given in spherical coordinates by $\rho = \sin\varphi$.



a. $\int_0^{\pi}\int_0^{2\pi}\int_0^{\sin\varphi} \rho^2\cos\varphi\,d\rho\,d\varphi\,d\theta$

b. $\int_0^{\pi}\int_0^{2\pi}\int_0^{1} \sin\varphi\,d\rho\,d\varphi\,d\theta$

c. $\int_0^{2\pi}\int_0^{\pi}\int_0^{\sin\varphi} \rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$ Correct Choice

d. $\int_0^{2\pi}\int_0^{\pi}\int_0^{1} \sin\varphi\,\rho^2\cos\varphi\,d\rho\,d\varphi\,d\theta$

e. $\int_0^{\pi}\int_0^{2\pi}\int_0^{\sin\varphi} 1\,d\rho\,d\varphi\,d\theta$

$$V = \iiint 1\,dV = \int_0^{2\pi}\int_0^{\pi}\int_0^{\sin\varphi} \rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$$

2. (10 points) Find the average temperature $T_{\text{ave}} = \dfrac{\iiint T\,dV}{\iiint dV}$ inside the region between the paraboloid $z = x^2 + y^2$ and the plane $z = 4$ if the temperature is given by $T = x^2z + y^2z$.

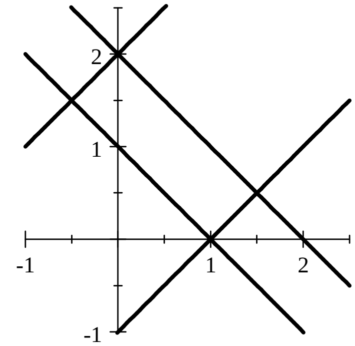
In cylindrical coordinates, $r^2 \le z \le 4$, $T = r^2z$ and $dV = r\,dr\,d\theta\,dz$

$$\iiint dV = \int_0^{2\pi}\int_0^{2}\int_{r^2}^{4} r\,dz\,dr\,d\theta = 2\pi\int_0^2 \Big[rz\Big]_{r^2}^4 dr = 2\pi\int_0^2 (4r - r^3)\,dr = 2\pi\left[2r^2 - \frac{r^4}{4}\right]_0^2 == 2\pi(8-4) =$$

$$\iiint T\,dV = \int_0^{2\pi}\int_0^{2}\int_{r^2}^{4} r^2zr\,dz\,dr\,d\theta = 2\pi\int_0^2 \left[r^3\frac{z^2}{2}\right]_{r^2}^4 dr = 2\pi\int_0^2 \left(8r^3 - \frac{r^7}{2}\right) dr = 2\pi\left[2r^4 - \frac{r^8}{16}\right]_0^2 == 2\pi$$

$$T_{\text{ave}} = \frac{32\pi}{8\pi} = 4$$

3. (10 points) Compute $\int\int (y-x)\,dx\,dy$ over the region bounded by the lines
$y = x-1,\quad y = x+2,\quad y = 1-x,$ and $y = 2-x.$



Use curvilinear coordinates.
Half credit for rectangular coordinates.

$u = y - x$
$v = y + x$

Boundaries: $u = -1 \quad u = 2$
$v = 1 \quad v = 2$

Solve for x and y:

$$u + v = 2y \qquad y = \frac{u+v}{2} \qquad x = \frac{v-u}{2}$$
$$v - u = 2x \qquad x = \frac{v-u}{2} \qquad y = \frac{u+v}{2}$$

Jacobian:

$$J = \left\| \begin{matrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{matrix} \right\| = \left\| \begin{matrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{matrix} \right\| = \left| -\frac{1}{4} - \frac{1}{4} \right| = \left| -\frac{1}{2} \right| = \frac{1}{2}$$

Integrand:

$$y - x = \frac{u+v}{2} - \frac{v-u}{2} = u$$

$$\int\int (y-x)\,dx\,dy = \int_1^2 \int_{-1}^2 u\,\frac{1}{2}\,du\,dv = \int_1^2 \frac{1}{2}\,\frac{u^2}{2}\Big|_{u=-1}^{2}\,dv = \int_1^2 \frac{3}{4}\,dv = \frac{3v}{4}\Big|_{v=1}^{2} = \frac{3}{4}$$