1. The vertices of a triangle are $P = (3, 4, -5)$, $Q = (3, 5, -4)$ and $R = (5, 2, -5)$. Find the angle at $P$.

   a. $90^\circ$
   b. $120^\circ$
   c. $135^\circ$
   d. $150^\circ$
   e. $180^\circ$

2. Find the volume of the parallelepiped with edge vectors:

   $\vec{a} = (4, 1, 2)$  $\vec{b} = (2, 2, 1)$  $\vec{c} = (1, 3, 0)$

   a. $-3$
   b. $0$
   c. $\sqrt{3}$
   d. $3$
   e. $9$
3. Consider the set of all points $P$ whose distance from $(1, 0, 0)$ is $3$ times its distance from $(-1, 0, 0)$. This set is a
   a. sphere.
   b. ellipsoid.
   c. hyperboloid.
   d. elliptic paraboloid.
   e. hyperbolic paraboloid.

4. For the curve $\mathbf{r}(t) = (\sin^2 t, \cos^2 t, \sin^2 t - \cos^2 t)$ which of the following is FALSE?
   a. $\mathbf{v} = (2 \sin t \cos t, -2 \sin t \cos t, 4 \sin t \cos t)$
   b. $|\mathbf{v}| = \sqrt{24} \sin t \cos t$
   c. $\mathbf{\hat{T}} = \left(\frac{2}{\sqrt{24}}, \frac{-2}{\sqrt{24}}, \frac{4}{\sqrt{24}}\right)$
   d. $ar = 0$
   e. $a_N = 0$

5. For the curve $\mathbf{r}(t) = (\sin^2 t, \cos^2 t, \sin^2 t - \cos^2 t)$ compute the arc length between $\mathbf{r}(0) = (0, 1, -1)$ and $\mathbf{r}\left(\frac{\pi}{2}\right) = (1, 0, 1)$.
   a. $\frac{1}{4} \sqrt{6}$
   b. $\frac{1}{2} \sqrt{6}$
   c. $\sqrt{6}$
   d. $2 \sqrt{6}$
   e. $4$
6. The plot at the right represents which vector field?
   a. \( \vec{A} = \langle x, y \rangle \)
   b. \( \vec{B} = \left( \frac{x}{\sqrt{x^2 + y^2}}, \frac{y}{\sqrt{x^2 + y^2}} \right) \)
   c. \( \vec{C} = \langle y, x \rangle \)
   d. \( \vec{D} = \left( \frac{y}{\sqrt{x^2 + y^2}}, \frac{x}{\sqrt{x^2 + y^2}} \right) \)
   e. \( \vec{E} = \langle x + y, x - y \rangle \)

7. Describe the level surfaces of \( f(x, y, z) = x^2 - y^2 - z^2 \).
   a. Elliptic Paraboloids
   b. Elliptic and Hyperbolic Paraboloids
   c. Hyperboloids of 1-sheet only
   d. Hyperboloids of 2-sheets only
   e. Hyperboloids of 1-sheet or 2-sheets

8. Find the plane tangent to the graph of \( z = xe^{xy} \) at the point \( (2, 0) \). Its \( z \)-intercept is
   a. 0
   b. 2
   c. \(-2\)
   d. 4
   e. \(-4\)
9. Find the plane tangent to the surface \( xyz + z^2 = 28 \) at the point \((4, 3, 2)\). Its \( z \)-intercept is
   a. 0
   b. 5
   c. \(-5\)
   d. 80
   e. \(-80\)

10. Find the line normal to the surface \( xyz + z^2 = 28 \) at the point \((4, 3, 2)\). It intersects the \( xy \)-plane at
   a. \((4, 3, 2)\)
   b. \((4, 3, 0)\)
   c. \(\left(\frac{13}{4}, 2, 0\right)\)
   d. \(\left(\frac{19}{4}, 4, 4\right)\)
   e. \(\left(\frac{19}{4}, 4, 0\right)\)

11. The salt concentration in a region of sea water is \( \rho = xy^2z^3 \). A swimmer is located at \((3, 2, 1)\). In what direction should the swimmer swim to increase the salt concentration as fast as possible?

   a. \(\langle 4, -12, 36 \rangle\)
   b. \(\langle -4, 12, -36 \rangle\)
   c. \(\langle 4, 12, 36 \rangle\)
   d. \(\langle -4, -12, -36 \rangle\)
   e. \(\langle 4, -12, -36 \rangle\)
Work Out: (12 points each. Part credit possible. Show all work.)

Do 4 of the following 5 problems. Cross out the one you do not want graded, here and on page 1. If you do not specify, #12 will be dropped.

12. Which of the following functions satisfy the Laplace equation \( f_{xx} + f_{yy} = 0 \)?

Show your work!

- a. \( f = x^2 + y^2 \)
- b. \( f = x^2 - y^2 \)
- c. \( f = x^3 + 3xy^2 \)
- d. \( f = x^3 - 3xy^2 \)
- e. \( f = e^{-x} \cos y + e^{-y} \cos x \)
- f. \( f = e^{-x} \cos y - e^{-y} \cos x \)

13. When two resistors with resistances \( R_1 \) and \( R_2 \) are connected in parallel, the net resistance \( R \) is given by

\[
\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R = \frac{R_1 R_2}{R_1 + R_2}.
\]

If \( R_1 \) and \( R_2 \) are measured as \( R_1 = 2 \pm 0.01 \) ohms and \( R_2 = 3 \pm 0.04 \) ohms, then \( R \) can be calculated as \( R = \frac{6}{5} \pm \Delta R \) ohms.

Use differentials to estimate the uncertainty \( \Delta R \) in the computed value of \( R \).
14. The average of a function $f$ on a curve $\vec{r}(t)$ is $f_{\text{ave}} = \frac{\int f \, ds}{\int ds}$.

Find the average of $f(x, y) = x^2$ on the circle $x^2 + y^2 = 9$.

HINTS: Parametrize the circle. $\sin^2 A = \frac{1 - \cos(2A)}{2}$, $\cos^2 A = \frac{1 + \cos(2A)}{2}$

15. A particle moves along the curve $\vec{r}(t) = (t^3, t^2, t)$ from $(1, 1, 1)$ to $(8, 4, 2)$ under the action of the force $\vec{F} = \langle z, y, x \rangle$. Find the work done.
16. The pressure in an ideal gas is given by \( P = k \rho T \) where \( k \) is a constant, \( \rho \) is the density and \( T \) is the temperature. At a certain instant, the measuring instruments are located at \( r_o = (1, 2, 3) \) and moving with velocity \( \vec{v} = \langle 4, 5, 6 \rangle \) and acceleration \( \vec{a} = \langle 7, 8, 9 \rangle \).

At that instant, the density and temperature are measured to be \( \rho = 12 \) and \( T = 300 \) and their gradients are \( \nabla \rho = \langle 0.6, 0.4, 0.2 \rangle \) and \( \nabla T = \langle 2, 1, 4 \rangle \).

Find \( \frac{dP}{dt} \), the time rate of change of the pressure as seen by the instruments.

Your answer may depend on \( k \).

HINTS: The pressure, \( P \) is a function of density, \( \rho \), and temperature, \( T \), which are functions of the position coordinates, \( (x, y, z) \), which are functions of time, \( t \). Use the chain rule.