1. For the curve $\vec{r}(t) = (t \cos t, t \sin t)$, which of the following is false?

   a. The velocity is $\vec{v} = (\cos t - t \sin t, \sin t + t \cos t)$
   
   b. The speed is $|\vec{v}| = \sqrt{1 + t^2}$
   
   c. The acceleration is $\vec{a} = (-2\sin t - t \cos t, 2\cos t - t \sin t)$
   
   d. The arclength between $t = 0$ and $t = 1$ is $L = \int_0^1 t \sqrt{1 + t^2} \, dt$
   
   e. The tangential acceleration is $a_T = \frac{t}{\sqrt{1 + t^2}}$

2. Find the plane tangent to the surface $x^2z^2 + y^4 = 5$ at the point $(2, 1, 1)$.

   a. $2x + y + z = 6$
   
   b. $2x + y + z = 5$
   
   c. $x + y + 2z = 5$
   
   d. $x - y + 2z = 3$
   
   e. $x - y + 2z = 6$
3. Let \( L = \lim_{(x,y) \to (0,0)} \frac{x^2 + xy^2}{x^2 + y^4} \)

a. \( L \) exists and \( L = 1 \) by looking at the paths \( y = mx \).

b. \( L \) does not exist by looking at the paths \( y = x \) and \( y = \sqrt{x} \).

c. \( L \) does not exist by looking at the paths \( y = \sqrt{x} \) and \( y = -\sqrt{x} \).

d. \( L \) does not exist by looking at the paths \( x = my^2 \).

e. \( L \) does not exist by looking at the paths \( x = y^3 \) and \( x = -y^3 \).

4. The point \((1, -3)\) is a critical point of the function \( f = xy^2 - 3x^3 + 6y \). It is a

a. local minimum.

b. local maximum.

c. saddle point.

d. inflection point.

e. The Second Derivative Test fails.

5. The dimensions of a closed rectangular box are measured as \( 70 \text{ cm}, 50 \text{ cm} \) and \( 40 \text{ cm} \) with a possible error of \( 0.2 \text{ cm} \) in each dimension. Use differentials to estimate the maximum error in the calculated surface area of the box.

a. 8

b. 16

c. 32

d. 64

e. 128
6. Consider the quarter of the cylinder \( x^2 + y^2 \leq 4 \) with \( x \geq 0, \ y \geq 0 \) and \( 0 \leq z \leq 8 \). Find the total mass of the quarter cylinder if the density is \( \rho = e^{x^2+y^2} \).

a. \( 2\pi(e^4 - 1) \)
b. \( 8\pi(e^4 - 1) \)
c. \( 2\pi e^4 \)
d. \( 8\pi e^4 \)
e. 4

7. Consider the quarter of the cylinder \( x^2 + y^2 \leq 4 \) with \( x \geq 0, \ y \geq 0 \) and \( 0 \leq z \leq 8 \). Find the \( z \)-component of the center of mass of the quarter cylinder if the density is \( \rho = e^{x^2+y^2} \).

a. \( 2\pi(e^4 - 1) \)
b. \( 8\pi(e^4 - 1) \)
c. \( 2\pi e^4 \)
d. \( 8\pi e^4 \)
e. 4

8. Compute the line integral \( \int y \, dx - x \, dy \) counterclockwise around the semicircle \( x^2 + y^2 = 9 \) from \( (3, 0) \) to \( (-3, 0) \). (HINT: Parametrize the curve.)

a. \(-9\pi\)
b. \(-3\pi\)
c. \(\pi\)
d. \(3\pi\)
e. \(9\pi\)
9. Compute the line integral \( \int \vec{F} \cdot d\vec{s} \) for the vector field \( \vec{F} = (y, x) \) along the curve \( \vec{r}(t) = (e^{\cos(t^2)}, e^{\sin(t^2)}) \) for \( 0 \leq t \leq \sqrt{\pi} \). (HINT: Find a scalar potential.)

a. \( e - \frac{1}{e} \)

b. \( \frac{1}{e} - e \)

c. \( \frac{2}{e} \)

d. \( 2e \)

e. \( 0 \)

10. Consider the parabolic surface \( P \) given by \( z = x^2 + y^2 \) for \( z \leq 4 \) with normal pointing up and in, the disk \( D \) given by \( x^2 + y^2 \leq 4 \) and \( z = 4 \) with normal pointing up, and the volume \( V \) between them. Given that for a certain vector field \( \vec{F} \) we have \( \iiint_V \nabla \cdot \vec{F} \, dV = 14 \) and \( \iint_D \vec{F} \cdot d\vec{S} = 3 \) compute \( \iint_P \vec{F} \cdot d\vec{S} \).

a. \( 17 \)

b. \( 11 \)

c. \( 8 \)

d. \( -11 \)

e. \( -17 \)
11. Find the dimensions and volume of the largest box which sits on the $xy$-plane and whose upper vertices are on the elliptic paraboloid $z + 2x^2 + 3y^2 = 12$.

You do not need to show it is a maximum.
You MUST use the Method of Lagrange multipliers.
Half credit for the Method of Eliminating the Constraint.
12. The hemisphere $H$ given by
\[ x^2 + y^2 + (z-2)^2 = 9 \quad \text{for} \quad z \geq 2 \]
has center $(0,0,2)$ and radius $3$. Verify Stokes' Theorem
\[ \iint_H \nabla \times \vec{F} \cdot d\vec{S} = \oint_{\partial H} \vec{F} \cdot d\vec{s} \]
for this hemisphere $H$ with normal pointing up and out
and the vector field $\vec{F} = (yz, -xz, z)$.

Be sure to check and explain the orientations. Use the following steps:

a. The hemisphere may be parametrized by
\[ \vec{R}(\theta, \varphi) = (3 \sin \varphi \cos \theta, 3 \sin \varphi \sin \theta, 2 + 3 \cos \varphi) \]
Compute the surface integral by successively finding:
\[ \vec{e}_{\theta}, \vec{e}_{\varphi}, \vec{N}, \nabla \times \vec{F}, \nabla \times \vec{F}(\vec{R}(\theta, \varphi)), \iint_H \nabla \times \vec{F} \cdot d\vec{S} \]
b. Parametrize the boundary circle $\partial H$ and compute the line integral by successively finding:
\[
\vec{r}(\theta), \quad \vec{v}(\theta), \quad \vec{F}(\vec{r}(\theta)), \quad \oint_{\partial H} \vec{F} \cdot d\vec{s}.
\]
Recall: $\vec{F} = (yz, -xz, z)$

13. The spider web at the right is the graph of the hyperbolic paraboloid $z = xy$.
It may be parametrized as
\[
\hat{R}(r, \theta) = (r \cos \theta, r \sin \theta, r^2 \sin \theta \cos \theta).
\]
Find the area of the web for $r \leq \sqrt{3}$. 
14. Green's Theorem states:
\[
\iint_R \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dx \, dy = \oint_{\partial R} P \, dx + Q \, dy
\]
Verify Green's Theorem for the functions
\[ P = -x^2y \quad \text{and} \quad Q = xy^2 \]
on the region inside the circle \( x^2 + y^2 = 16 \).
Use the following steps:

a. Compute \( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \).
Then compute \( \iint_R \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dx \, dy \) by converting to polar coordinates.

b. Parametrize the boundary circle.
Compute \( P, \ Q, \ dx \) and \( dy \) on the boundary curve.
Then compute \( \oint_{\partial R} P \, dx + Q \, dy \) around the boundary.