Name__________________________ ID________________

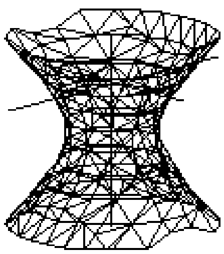
MATH 251 Quiz 2 Fall 2006

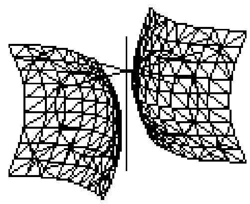
Sections 507 Solutions P. Yasskin

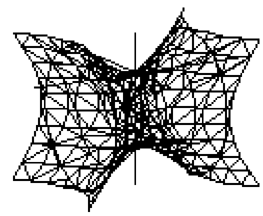
1-5	/25

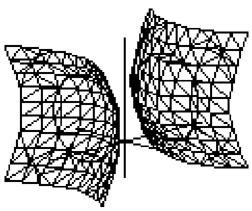
Multiple Choice (5 points each)

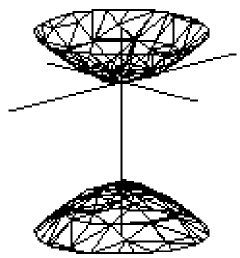
1. Which of the following is the graph of the quadric surface $x^2 - y^2 - z^2 - 2z - 2 = 0$?

a.


c.

Correct Choice

e.


b.


d.


Complete square: $x^2 - y^2 - (z+1)^2 = 1$

Hyperboloid of 2 sheets with axis parallel to x-axis thru $(0,0,-1)$.

2. The plot at the right represents which vector field?

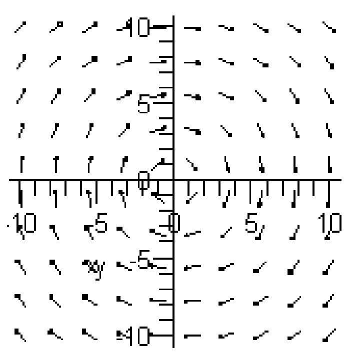
a. $\vec{A} = \langle -y, x \rangle$

b. $\vec{B} = \left\langle \dfrac{-y}{\sqrt{x^2+y^2}}, \dfrac{x}{\sqrt{x^2+y^2}} \right\rangle$

c. $\vec{C} = \langle y, -x \rangle$

d. $\vec{D} = \left\langle \dfrac{y}{\sqrt{x^2+y^2}}, \dfrac{-x}{\sqrt{x^2+y^2}} \right\rangle$ Correct Choice

e. $\vec{E} = \langle x, -y \rangle$



The vectors all have the same length and aim clockwise.

$\vec{A}, \vec{C}$ and $\vec{E}$ all have length $\sqrt{x^2+y^2}$. $\vec{A}$ and $\vec{B}$ are clockwise.

3. Compute $\int x\,ds$ along the curve $\vec{r}(t) = \langle t, t^2, t^2 \rangle$ between $(0,0,0)$ and $(1,1,1)$.

a. $\frac{13}{12}$ Correct Choice

b. $\frac{9}{8}$

c. $\frac{13}{8}$

d. $\frac{27}{16}$

e. $\frac{9}{16}$

$\vec{v}(t) = \langle 1, 2t, 2t \rangle \qquad |\vec{v}| = \sqrt{1+4t^2+4t^2} = \sqrt{1+8t^2} \qquad ds = |\vec{v}|\,dt = \sqrt{1+8t^2}\,dt$

$(0,0,0) = \vec{r}(0) \qquad (1,1,1) = \vec{r}(1)$

$$\int x\,ds = \int_0^1 t\sqrt{1+8t^2}\,dt = \frac{(1+8t^2)^{3/2}}{24}\Bigg|_0^1 = \frac{27}{24} - \frac{1}{24} = \frac{13}{12}$$

4. Compute $\int yz\,dx + xz\,dy + xy\,dz$ along the curve $\vec{r}(t) = \langle t, t^2, t^2 \rangle$ between $(0,0,0)$ and $(1,1,1)$.

a. 0

b. 1 Correct Choice

c. 2

d. 3

e. 4

Method 1: $\vec{v}(t) = \langle 1, 2t, 2t \rangle \qquad \vec{F} = \langle yz, xz, xy \rangle = \langle t^4, t^3, t^3 \rangle$

$$\int yz\,dx + xz\,dy + xy\,dz = \int \vec{F}\cdot d\vec{s} = \int \vec{F}\cdot\vec{v}\,dt = \int (t^4 + 2t^4 + 2t^4)\,dt = \int 5t^4\,dt = t^5\big|_0^1 = 1$$

Method 2: $x = t, \quad y = t^2, \quad z = t^2 \qquad dx = dt, \quad dy = 2t\,dt, \quad dz = 2t\,dt$

$$\int yz\,dx + xz\,dy + xy\,dz = \int (t^4\,dt + t^3 2t\,dt + t^3 2t\,dt) = \int 5t^4\,dt = t^5\big|_0^1 = 1$$

5. Compute $\int \vec{F}\cdot d\vec{s}$ for $\vec{F} = \left\langle \frac{-y}{\sqrt{x^2+y^2}}, \frac{x}{\sqrt{x^2+y^2}} \right\rangle$ once counterclockwise around the circle $x^2 + y^2 = 4$.

a. $\frac{\pi}{4}$

b. $\frac{\pi}{2}$

c. π

d. 2π

e. 4π Correct Choice

$\vec{r}(t) = \langle 2\cos(t), 2\sin(t) \rangle \qquad \vec{v}(t) = \langle -2\sin(t), 2\cos(t) \rangle \qquad \vec{F}(\vec{r}(t)) = \left\langle \frac{-2\sin(t)}{2}, \frac{2\cos(t)}{2} \right\rangle = \langle -\sin(t), \cos(t) \rangle$

$$\int \vec{F}\cdot d\vec{s} = \int \vec{F}\cdot\vec{v}\,dt = \int_0^{2\pi} (2\sin^2(t) + 2\cos^2(t))\,dt = \int_0^{2\pi} 2\,dt = 2t\big|_0^{2\pi} = 4\pi$$