Multiple Choice: (5 points each)

1. For the function $f(x, y) = y^2 \cos(xy)$ which partial derivative is incorrect?

   a. $\frac{\partial f}{\partial x} = -y^3 \sin(xy)$
   
   b. $\frac{\partial f}{\partial y} = 2y \cos(xy) - xy^2 \sin(xy)$
   
   c. $\frac{\partial^2 f}{\partial x^2} = -y^4 \cos(xy)$
   
   d. $\frac{\partial^2 f}{\partial y \partial x} = -3y^2 \sin(xy) - xy^3 \cos(xy)$
   
   e. $\frac{\partial^2 f}{\partial x \partial y} = -y^2 \sin(xy) - xy^3 \cos(xy)$  Correct Choice

   Use product rule and chain rule:
   
   \[
   \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} (2y \cos(xy) - xy^2 \sin(xy)) = -2y^2 \sin(xy) - y^2 \sin(xy) - xy^3 \cos(xy)
   \]

2. Find the equation of the plane tangent to $z = x^2 y^3$ at the point $(2, 1, 4)$. Its $z$-intercept is:

   a. 0
   
   b. −24
   
   c. −16  Correct Choice
   
   d. 24
   
   e. 4

   $f(x, y) = x^2 y^3$  \quad $f(2, 1) = 4$

   $\frac{\partial f}{\partial x} = 2xy^3$  \quad $\frac{\partial f}{\partial x}(2, 1) = 4$

   $\frac{\partial f}{\partial y} = 3x^2 y^2$  \quad $\frac{\partial f}{\partial y}(2, 1) = 12$

   $z = f_{\text{tan}}(x, y) = f(2, 1) + \frac{\partial f}{\partial x}(2, 1)(x - 2) + \frac{\partial f}{\partial y}(2, 1)(y - 1) = 4 + 4(x - 2) + 12(y - 1)$

   $z = 4x + 12y - 16$  The $z$-intercept is −16
3. Consider a function $p(x, y)$. If $p(2, 3) = 3$, $\frac{\partial p}{\partial x}(2, 3) = 4$, and $\frac{\partial p}{\partial y}(2, 3) = 5$, estimate $p(2.1, 2.8)$.

a. 2.4 Correct Choice
b. 2.6
c. 2.8
d. 3.2
e. 3.4

$p_{\text{tan}}(x, y) = p(2, 3) + \frac{\partial p}{\partial x}(2, 3)(x - 2) + \frac{\partial p}{\partial y}(2, 3)(y - 3) = 3 + 4(x - 2) + 5(y - 3)$

$p(3.2, 1.9) = p_{\text{tan}}(2.1, 2.8) = 3 + 4(2.1 - 2) + 5(2.8 - 3) = 3 + 4(0.1) + 5(-0.2) = 2.4$

4. If the temperature in a room is given by $T = 75 + xyz$ and a fly is located at $(2, 1, 4)$, in what unit vector direction should the fly fly in order to decrease the temperature as fast as possible?

a. $\frac{1}{\sqrt{21}}(2, 4, 1)$
b. $\frac{1}{\sqrt{21}}(-2, -4, -1)$ Correct Choice
c. $(4, 8, 2)$
d. $(-4, -8, -2)$
e. $\frac{1}{\sqrt{21}}(2, -4, 1)$

$\nabla T = \langle yz, xz, xy \rangle$ \quad $\vec{v} = \nabla T|_{(2,1,4)} = \langle 4, 8, 2 \rangle$ \quad $|\vec{v}| = \sqrt{16 + 64 + 4} = \sqrt{84} = 2\sqrt{21}$

Direction of Max increase is $\hat{v} = \frac{\vec{v}}{|v|} = \frac{1}{\sqrt{21}}(2, 4, 1)$.

Direction of Max decrease is $-\hat{v} = -\frac{1}{\sqrt{21}}(2, 4, 1)$.

5. Find the equation of the plane tangent to the surface $x^2z^2 + xy^3 = 31$ at the point $(1, 3, 2)$. Its $z$-intercept is:

a. $-31$
b. 124
c. 120
d. 31 Correct Choice
e. 4

$P = (1, 3, 2)$ \quad $F = x^2z^2 + xy^3$ \quad $\vec{V}F = \langle 2xz^2 + y^3, 3xy^2, 2x^2z \rangle$

$\vec{N} = \vec{V}F|_P = \langle 2 \cdot 1 \cdot 4 + 27, \ 3 \cdot 1 \cdot 9, \ 2 \cdot 1 \cdot 2 \rangle = \langle 35, 27, 4 \rangle$

Tangent plane is $\vec{N} \cdot X = \vec{N} \cdot P$ or $35x + 27y + 4z = 35 \cdot 1 + 27 \cdot 3 + 4 \cdot 2 = 124$

or $z = 31 - \frac{35}{4}x - \frac{27}{4}y$ The $z$-intercept is 31.