1. Find the equation of the sphere which passes through the points \((2, 1, 3)\) and \((4, 5, -1)\).
   
   a. \((x - 3)^2 + (y - 3)^2 + (z - 1)^2 = 3\)
   
   b. \((x - 3)^2 + (y - 3)^2 + (z - 1)^2 = 9\) Correct Choice
   
   c. \((x + 3)^2 + (y + 3)^2 + (z + 1)^2 = 9\)
   
   d. \((x - 3)^2 + (y - 3)^2 + (z - 1)^2 = 36\)
   
   e. \((x + 3)^2 + (y + 3)^2 + (z + 1)^2 = 36\)

   The center is the midpoint: \(\mathbf{C} = \frac{\mathbf{P} + \mathbf{Q}}{2} = (3, 3, 1)\)

   The diameter is the distance: \(d(\mathbf{P}, \mathbf{Q}) = \sqrt{2^2 + 4^2 + 4^2} = 6\)

   The radius is half the diameter: \(r = 3\)

   The circle is: \((x - 3)^2 + (y - 3)^2 + (z - 1)^2 = 9\)

2. The angle between the vectors \(\mathbf{u} = (2, -2, 4)\) and \(\mathbf{v} = (-1, 2, 1)\) is
   
   a. acute
   
   b. right
   
   c. obtuse Correct Choice
   
   d. none of these

   Since \(\mathbf{u} \cdot \mathbf{v} = -2 - 4 + 4 = -2 < 0\), the angle is obtuse.

3. A wagon is pulled horizontally from the origin \((0, 0)\) to the point \((5, 0)\) meters by the force \(\mathbf{F} = (4, 3)\) Newtons. Find the work done.
   
   a. 20 Joules Correct Choice
   
   b. 15 Joules
   
   c. 25 Joules
   
   d. \(5\sqrt{5}\) Joules
   
   e. \(\sqrt{5}\) Joules

   The displacement vector is \(\mathbf{D} = (5, 0)\). So the work is \(W = \mathbf{F} \cdot \mathbf{D} = 10\) Joules.
4. A triangle has vertices $P = (-1, 2, -3)$, $Q = (3, 2, 1)$, and $R = (-1, -1, 0)$. Find the angle at $P$.

   a. $30^\circ$
   b. $45^\circ$
   c. $60^\circ$ Correct Choice
   d. $90^\circ$
   e. $120^\circ$

   $\overrightarrow{PQ} = Q - P = (4, 0, 4)$  
   $\overrightarrow{PR} = R - P = (0, -3, 3)$

   $|\overrightarrow{PQ}| = \sqrt{16 + 16} = 4\sqrt{2}$  
   $|\overrightarrow{PR}| = \sqrt{9 + 9} = 3\sqrt{2}$  
   $\overrightarrow{PQ} \cdot \overrightarrow{PR} = 12$

   $\cos \theta = \frac{12}{4\sqrt{2} \cdot 3\sqrt{2}} = \frac{1}{2}$  
   $\theta = 60^\circ$ (Use a 30-60-90 triangle.)

5. A triangle has vertices $P = (-1, 2, -3)$, $Q = (3, 2, 1)$, and $R = (-1, -1, 0)$. Find the projection of the side $\overrightarrow{PQ}$ onto the base $\overrightarrow{PR}$.

   a. $\text{proj}_{\overrightarrow{PR}} \overrightarrow{PQ} = \left( \frac{3}{2}, 0, \frac{3}{2} \right)$
   b. $\text{proj}_{\overrightarrow{PR}} \overrightarrow{PQ} = (-1, -1, 0)$
   c. $\text{proj}_{\overrightarrow{PR}} \overrightarrow{PQ} = \left( -\frac{2}{3}, -\frac{2}{3}, 0 \right)$
   d. $\text{proj}_{\overrightarrow{PR}} \overrightarrow{PQ} = (0, -3, 3)$
   e. $\text{proj}_{\overrightarrow{PR}} \overrightarrow{PQ} = (0, -2, 2)$ Correct Choice

   $\overrightarrow{PQ} = Q - P = (4, 0, 4)$  
   $\overrightarrow{PR} = R - P = (0, -3, 3)$

   $|\overrightarrow{PR}|^2 = 9 + 9 = 18$  
   $\overrightarrow{PQ} \cdot \overrightarrow{PR} = 12$

   $\text{proj}_{\overrightarrow{PR}} \overrightarrow{PQ} = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{|\overrightarrow{PR}|^2} \overrightarrow{PR} = \frac{12}{18} (0, -3, 3) = (0, -2, 2)$