Name____________________________ Sec__________

1-11	/55	14	/20
12	/10	15	/11
13	/10	Total	/106

MATH 251 Honors Exam 1 Spring 2010

Sections 200 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. The points $A = (2,-3.4)$ and $B = (4,1,0)$ are the endpoints of the diameter of a sphere. What is the radius of the sphere?

a. 2

b. 3

c. 4

d. 5

e. 6

2. Find a vector perpendicular to the plane containing the points

$$P = (2,1,4), \quad Q - (-1,3,2) \quad \text{and} \quad R = (3,1,2)$$

a. $(2,-1,2)$

b. $(-4,8,-2)$

c. $(2,4,1)$

d. $(2,-2,1)$

e. $(-4,2,-4)$

3. Find the angle between the normals to the planes $3x+2y-4z = 3$ and $2x-y+z = 2$.

a. $0°$

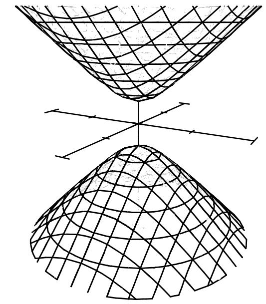
b. $30°$

c. $45°$

d. $60°$

e. $90°$

4. The plot at the right is the graph of which equation?



a. $x^2 + y^2 - z^2 = 1$

b. $x^2 + y^2 - z^2 = 0$

c. $x^2 + y^2 - z^2 = -1$

d. $x^2 + y^2 - z = 1$

e. $x^2 + y^2 - z = -1$

5. For the curve $\vec{r}(t) = \left(e^{-t}, \sqrt{2}\,t, e^{t}\right)$ between $A = (1,0,1)$ and $B = \left(e^{-2}, 2\sqrt{2}, e^{2}\right)$, which of the following is FALSE?

a. $\vec{v} = \left(-e^{-t}, \sqrt{2}, e^{t}\right)$ velocity

b. $\vec{a} = (e^{-t}, 0, e^{t})$ acceleration

c. $\dfrac{ds}{dt} = e^{-t} + e^{t}$ speed

d. $a_T = e^{-t} - e^{t}$ tangential acceleration

e. $L = e^{2} - e^{-2}$ arc length between A and B

6. Compute $\int_A^B \vec{F} \cdot d\vec{s}$ with $\vec{F} = (-z, y, x)$ along the curve $\vec{r}(t) = \left(e^{-t}, \sqrt{2}\,t, e^{t}\right)$ between $A = (1,0,1)$ and $B = \left(e^{-2}, 2\sqrt{2}, e^{2}\right)$.

a. $e^{4} - e^{-4} - 1$

b. $e^{2} - e^{-2} - 1$

c. $e^{4} - e^{-4} - 2$

d. $e^{2} - e^{-2} - 2$

e. 8

7. If $f(x,y,z) = y^2z^2 + x^2\sin(yz)$, which of the following is $\dfrac{\partial^3 f}{\partial z\partial y\partial x}$?

a. $-2xyz\sin(yz)$

b. $2x\cos(yz) - 2xyz\sin(yz)$

c. $4yz - 2xyz\sin(yz)$

d. $2yz + 2x\cos(yz) - 2xyz\sin(yz)$

e. $4yz + 2x\cos(yz) - 2xyz\sin(yz)$

8. Find the plane tangent to the graph of the function $z = x^2y^3$ at the point $(x,y) = (3,2)$.
What is the z-intercept?

a. -288

b. -144

c. -72

d. 72

e. 144

9. Find the plane tangent to the graph of the equation $xy + xz + yz = 11$ at the point $(x,y,z) = (3,2,1)$.

What is the z-intercept?

a. $-\dfrac{11}{5}$

b. $\dfrac{11}{5}$

c. $\dfrac{22}{5}$

d. 11

e. 22

10. The pressure in a certain ideal gas is given by $P = \dfrac{T}{100V}$ where the temperature is currently $T = 300°K$ and increasing at $2°K/\text{min}$ and the volume is currently $V = 4\ \text{m}^3$ and increasing at $\dfrac{1}{3}\ \text{m}^3/\text{min}$. Is the pressure increasing or decreasing and at what rate?

a. decreasing at $\dfrac{23}{400}$ atm/min

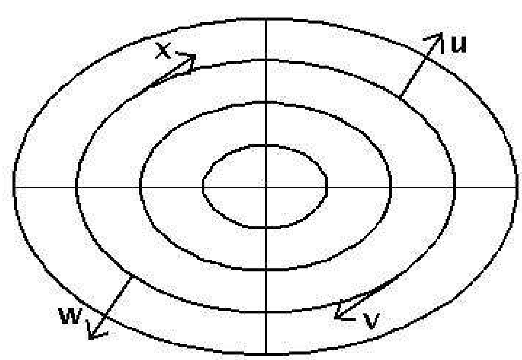
b. decreasing at $\dfrac{27}{400}$ atm/min

c. increasing at $\dfrac{23}{400}$ atm/min

d. increasing at $\dfrac{25}{400}$ atm/min

e. increasing at $\dfrac{27}{400}$ atm/min

11. The graph at the right shows the contour plot of a function $f(x,y)$ as well as several vectors. Which vectors could not be the gradient of f?



a. $\vec{u}$ and $\vec{v}$

b. $\vec{u}$ and $\vec{w}$

c. $\vec{v}$ and $\vec{w}$

d. $\vec{v}$ and $\vec{x}$

e. $\vec{w}$ and $\vec{x}$

Work Out: (Points indicated. Part credit possible. Show all work.)

12. (10 points) A cardboard box is 5 inches long, 4 inches wide and 3 inches high.

The cardboard is 0.05 inches thick.

Use differentials to estimate the volume of cardboard used to make the box.

13. (10 points) A wire has the shape of the curve $\vec{r}(t) = \left(e^{-t}, \sqrt{2}\,t, e^{t}\right)$ between $A = (1,0,1)$ and $B = \left(e^{-2}, 2\sqrt{2}, e^{2}\right)$. (See problem 5.) Find its mass if its linear density is given by $\rho = z - x$.

14. (20 points) Duke Skywater is chasing the Dark Invader through a Dark Matter field. Duke is currently at the point $P = (3,2,1)$ and the dark matter density is $\rho = xy + xz + yz$.

a. What is the time rate of change of the dark matter density as seen by Duke if his velocity is $\vec{v} = (1,2,3)$?

b. In what **unit** vector direction should Duke travel to increase the dark matter density as fast as possible?

c. What is the maximum rate of increase of the dark matter density in any unit vector direction?

15. (11 points) For each of the following limits, say whether or not the limit exists.

If it exists, give its value and prove it.

If it does not exist, give a counter example.

a. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$

b. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4+x^2y^4}{x^2+y^4}$

c. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^2+y^2}$