Name________________________ Sec__________

MATH 251 Exam 2 Spring 2010

Sections 511 P. Yasskin

1-10	/60	12	/15
11	/10	13	/20
		Total	/105

Multiple Choice: (6 points each. No part credit.)

1. The point $(2,1)$ is a critical point of the function $f(x,y) = x^3 - 3x^2 + xy^2 - 4xy + 3x + 4y$.
Use the Second Derivative Test to classify this critical point.

a. Local Maximum

b. Local Minimum

c. Inflection Point

d. Saddle Point

e. Test Fails

2. Compute $\int_0^1 \int_0^x \int_0^y xy\,dz\,dy\,dx$

a. $\frac{1}{12}$

b. $\frac{1}{15}$

c. $\frac{1}{20}$

d. $\frac{2}{15}$

e. $\frac{3}{20}$

3. Find the mass of the tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,2,0)$ and $(0,0,4)$, if the density is $\rho = x$.

a. $\frac{1}{3}$

b. $\frac{4}{3}$

c. $\frac{8}{3}$

d. 3

e. 4

4. Find the volume of the solid below $z = 4 - x^2 - y^2$ above the xy-plane.

a. π

b. 2π

c. 4π

d. 8π

e. 16π

5. Find the average of the function $f = z^2$ within the solid below $z = 4 - x^2 - y^2$ above the xy-plane.

a. $\frac{4}{5}$

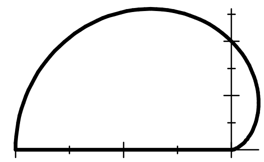
b. $\frac{8\pi}{5}$

c. $\frac{8}{3}$

d. $\frac{16}{3}$

e. $\frac{64\pi}{3}$

6. Find the volume below $z = y$ above the region between the x-axis and the upper half of the cardioid $r = 1 - \cos\theta$.



a. $\frac{1}{12}$

b. $\frac{1}{6}$

c. $\frac{2}{3}$

d. $\frac{4}{3}$

e. $\frac{8}{3}$

7. Compute $\iiint e^{\left(x^2+y^2+z^2\right)^{3/2}}\,dV$ over the hemisphere $x^2 + y^2 + z^2 \le 9$ with $z \ge 0$.

a. $\frac{\pi}{3}(e^{27} - 1)$

b. $\frac{\pi}{3}e^{27}$

c. $\frac{2\pi}{3}(e^{27} - 1)$

d. $\frac{2\pi}{3}e^{27}$

e. $\frac{4\pi}{3}(e^{27} - 1)$

8. Compute $\oint \vec{F} \cdot d\vec{s}$ for the vector field $\vec{F} = (-yz^2, xz^2, z^3)$ around the circle $\vec{r}(\theta) = (2\cos\theta, 2\sin\theta, 8)$.

a. -256π

b. -128π

c. 128π

d. 256π

e. 512π

9. Find the area of the piece of the surface $z = xy$ for (x,y) in the semicircle $x^2 + y^2 \le 9$ for $y \ge 0$.

HINT: Parametrize the surface as $\vec{R}(u,v) = (u,v,???)$.

a. $\frac{\pi}{3}(10^{3/2} - 1)$

b. $\frac{2\pi}{3}(10^{3/2} - 1)$

c. 9π

d. 18π

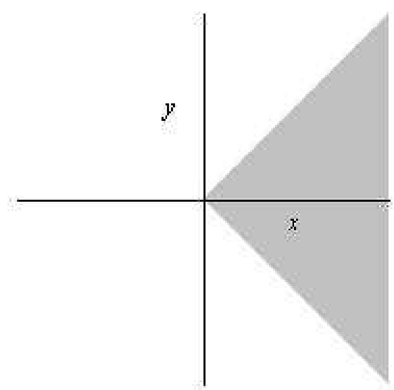
e. 36π

10. The hyperbolic coordinate system given by

$$x = \frac{r}{2}(e^s + e^{-s}) \qquad y = \frac{r}{2}(e^s - e^{-s})$$

covers the quarter plane shown.

Find its Jacobian factor.



a. $r(e^s + e^{-s})$

b. $r\left|e^s - e^{-s}\right|$

c. $\frac{r}{2}(e^{2s} + e^{-2s})$

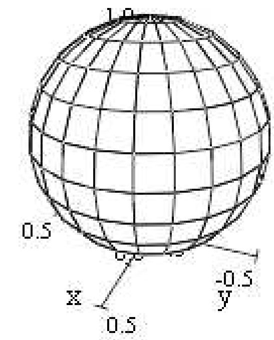
d. $\frac{r}{2}|e^{2s} - e^{-2s}|$

e. r

Work Out: (Points indicated. Part credit possible. Show all work.)

11. (10 points) Find the dimensions and volume of the largest rectangular solid box which sits on the xy-plane and has its upper vertices on the paraboloid $z = 4 - 4x^2 - y^2$.

12. (15 points) Find the mass and center of mass of the solid sphere of radius $\frac{1}{2}$ sitting on the xy-plane, given in spherical coordinates by $\rho = \cos\varphi$, if the mass density is $\delta = z$.



13. (20 points) Compute $\iint_P \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ for the vector field $\vec{F} = (-yz^2, xz^2, z^3)$ over the paraboloid $z = 2x^2 + 2y^2$ for $z \le 8$ oriented down and out.
Follow these steps. Be sure to check the orientation.

$\vec{\nabla} \times \vec{F} =$

Parametrize the surface as $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, 2r^2)$

$\vec{e}_r =$

$\vec{e}_\theta =$

$\vec{N} =$

$\left(\vec{\nabla} \times \vec{F}\right)\left(\vec{R}(r,\theta)\right) =$

$\left(\vec{\nabla} \times \vec{F}\right) \cdot \vec{N} =$

$\iint_P \vec{\nabla} \times \vec{F} \cdot d\vec{S} =$