Name________________________ Sec__________

MATH 251 Final Spring 2010

Sections 511 P. Yasskin

1-13	/65	15	/10
14	/21	16	/10
		Total	/106

Multiple Choice: (5 points each. No part credit.)

1. Find the angle between the vectors $\vec{u} = (2, 2, 1)$ and $\vec{v} = (1, 2, 2)$.

a. $\arccos(8/9)$

b. $\arccos(8/3)$

c. $\arccos\left(\sqrt{8}/9\right)$

d. $\arccos\left(3/\sqrt{8}\right)$

e. $\arccos(3/8)$

2. At the point (x, y, z) where the line $\vec{r}(t) = (1 - t, t, 2 - 2t)$ intersects the plane $x - 2y + 3z = 16$, we have $x + y + z =$

a. -2

b. 2

c. 3

d. 5

e. 16

3. If a jet flies around the world from West to East, directly above the equator, in what direction does its unit binormal $\hat{B}$ point?

a. Down (toward the center of the earth)

b. Up (away from the center of the earth)

c. North

d. South

e. West

4. Find the z-intercept of the plane tangent to the surface $\frac{xy}{z} = 1$ at the point $(2, 3, 6)$.

a. 6

b. $\frac{1}{6}$

c. 5

d. -5

e. -6

5. The temperature in an ideal gas is given by $T = \kappa \frac{P}{\rho}$ where κ is a constant, P is the pressure and ρ is the density. At a certain point $Q = (3, 2, 1)$, we have

$$P(Q) = 8 \qquad \vec{\nabla} P(Q) = (4, -2, -4)$$

$$\rho(Q) = 2 \qquad \vec{\nabla} \rho(Q) = (-1, 4, 2)$$

So at the point Q, the temperature is $T(Q) = 4\kappa$ and its gradient is $\vec{\nabla} T(Q) =$

a. $\kappa(-8.5, 6, 9)$

b. $\kappa(4, -9, -6)$

c. $\kappa(3, 2, -2)$

d. $\kappa\left(\frac{1}{2}, 2\right)$

e. $\kappa\left(-\frac{1}{2}, 2\right)$

6. If the temperature in a room is $T = xyz^2$, find the rate of change of the temperature as seen by a fly who is located at $(3, 2, 1)$ and has velocity $(1, 2, 3)$.

a. 32

b. 36

c. 44

d. 48

e. 52

7. Find the volume below $z = xy$ above the region between the curves $y = 3x$ and $y = x^2$.

a. $\frac{81}{2}$

b. $\frac{81}{4}$

c. $\frac{81}{8}$

d. $\frac{243}{2}$

e. $\frac{243}{8}$

8. Compute $\iint\limits_{C} e^{-x^2-y^2}\,dx\,dy$ over the disk enclosed in the circle $x^2 + y^2 = 4$.

a. $\frac{\pi}{2}(1 - e^{-4})$

b. $\pi(1 - e^{-4})$

c. $\frac{\pi}{2}e^{-4}$

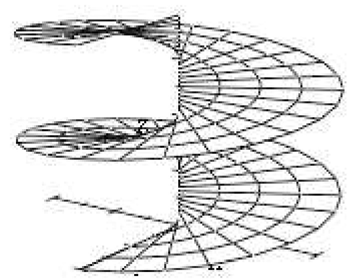
d. πe^{-4}

e. $2\pi e^{-4}$

9. Find the mass of **2 loops** of the helical ramp parametrized by

$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, 4\theta) \quad \text{for} \quad r \le 3$$

if the density is $\rho = \sqrt{x^2 + y^2}$.



a. 40π

b. 120π

c. 200π

d. $\frac{500}{3}\pi$

e. $\frac{244}{3}\pi$ Correct Choice

10. Find the flux of $\vec{F} = (y, -x, 2)$ through the helical ramp of problem 9 oriented up.

a. 4π

b. $\frac{8}{3}\pi$

c. 216π

d. 108π Correct Choice

e. $\frac{1024}{3}\pi$

11. Compute $\int_{(2,1)}^{(3,2)} \vec{F} \cdot d\vec{s}$ for $\vec{F} = (2xy, x^2)$ along the curve $\vec{r}(t) = ((2+t^2)e^{\sin \pi t}, (1+t^2)e^{\sin 2\pi t})$.
HINT: Find a scalar potential.

a. 12

b. 14

c. 22

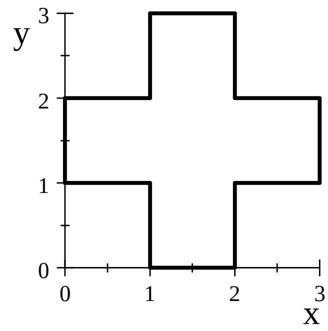
d. $\sqrt{2}$

e. $15 - 4\sqrt{2}$

12. Compute $\oint (2x\sin y - 5y)\,dx + (x^2\cos y - 4x)\,dy$

counterclockwise around the cross shown.

HINT: Use Green's Theorem.



a. -45

b. -10

c. 5

d. 10

e. 45

13. Compute $\iint_S \vec{\nabla}\times\vec{F}\cdot d\vec{S}$ for $\vec{F} = (-yz, xz, xyz)$

over the quartic surface $z = (x^2+y^2)^2$ for $z \le 16$

oriented down and out. The surface may be parametrized by

$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^4)$$

HINT: Use Stokes' Theorem.

a. -128π

b. -64π

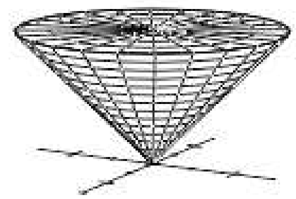
c. -32π

d. 32π

e. 64π

Work Out: (Points indicated. Part credit possible. Show all work.)

14. (21 points) Verify Gauss' Theorem $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV = \iint\limits_{\partial V} \vec{F} \cdot d\vec{S}$



for the vector field $\vec{F} = (4xz^3, 4yz^3, z^4)$ and the solid V
above the cone C given by $z = \sqrt{x^2 + y^2}$
or parametrized by $R(r,\theta) = (r\cos\theta, r\sin\theta, r)$,
below the disk D given by $x^2 + y^2 \le 9$ and $z = 3$.
Be sure to check and explain the orientations.
Use the following steps:

a. (4 pts) Compute the volume integral by successively finding:

$$\vec{\nabla} \cdot \vec{F}, \quad dV, \quad \iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV$$

b. (8 pts) Compute the surface integral over the disk by parametrizing the disk and successively finding:

$$\vec{R}(r,\theta), \quad \vec{e}_r, \quad \vec{e}_\theta, \quad \vec{N}, \quad \vec{F}\left(\vec{R}(r,\theta)\right), \quad \iint\limits_D \vec{F} \cdot d\vec{S}$$

Recall: $\vec{F} = (4xz^3, 4yz^3, z^4)$ and C is the cone parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$.

c. (7 pts) Compute the surface integral over the cone C by successively finding:

$$\vec{e}_r,\quad \vec{e}_\theta,\quad \vec{N},\quad \vec{F}\left(\vec{R}(r,\theta)\right),\quad \iint_C \vec{F} \cdot d\vec{S}$$

d. (2 pts) Combine $\iint_D \vec{F} \cdot d\vec{S}$ and $\iint_C \vec{F} \cdot d\vec{S}$ to get $\iint_{\partial V} \vec{F} \cdot d\vec{S}$

15. (10 points) Find the average value of the function $f(x,y,z) = x^2 + y^2 + z^2$ within the solid cylinder $x^2 + y^2 \le 9$ for $0 \le z \le 4$.

16. (10 points) Find the value(s) of R so that the ellipsoid $\frac{x^2}{4^2} + \frac{y^2}{3^2} + \frac{z^2}{2^2} = R^2$ is tangent to the plane $\frac{1}{2}x + \frac{4}{3}y + 2z = 36$.

HINT: Their normal vectors must be parallel.