Name______________________________ Sec__________

MATH 251H Exam 2 Spring 2011

Section 200 P. Yasskin

1-12	/60	14	/15
13	/15	15	/15
		Total	/105

Multiple Choice: (5 points each. No part credit.)

1. Compute $\int_0^2 \int_0^y xy\,dx\,dy$.

a. 1
b. 2
c. 3
d. 4
e. y^2

2. Find the area of one loop of the rose $r = \sin(3\theta)$.

a. $\frac{\pi}{12}$
b. $\frac{\pi}{6}$
c. $\frac{\pi}{4}$
d. $\frac{\pi}{3}$
e. $\frac{\pi}{2}$

3. Compute $\iiint x^2 + y^2\,dV$ over the region between the cones $z = \sqrt{x^2+y^2}$ and $z = 4 - \sqrt{x^2+y^2}$.

a. $\frac{8\pi}{3}$
b. $\frac{16\pi}{3}$
c. $\frac{32\pi}{3}$
d. $\frac{16\pi}{5}$
e. $\frac{32\pi}{5}$

4. Find the mass of the hemisphere $x^2 + y^2 + z^2 \le 4$ with $y \ge 0$ if the density is $\delta = y$.

a. $\frac{\pi}{2}$
b. π
c. 2π
d. 4π
e. 8

5. Find the center of mass of the hemisphere $x^2 + y^2 + z^2 \le 4$ with $y \ge 0$ if the density is $\delta = y$.

a. $\left(0, \frac{64\pi}{15}, 0\right)$
b. $\left(0, \frac{16}{15}, 0\right)$
c. $\left(0, \frac{\pi^2}{12}, 0\right)$
d. $\left(0, \frac{15}{16}, 0\right)$
e. $\left(0, \frac{12}{\pi^2}, 0\right)$

6. Compute $\oint \vec{F} \cdot d\vec{s}$ for $\vec{F} = (-16x^2y, 9xy^2)$ counterclockwise around the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1.$

HINTS: The ellipse may be parametrized by $\vec{r}(\theta) = (3\cos\theta, 4\sin\theta).$
Since $\sin(2\theta) = 2\sin\theta\cos\theta,$ we have $4\sin^2\theta\cos^2\theta = \sin^2(2\theta).$

a. -864π
b. -288π
c. 144π
d. 288π
e. 864π

7. The point $\left(\frac{\pi}{3}, \frac{\pi}{6}\right)$ is a critical point of the function $f(x,y) = \sin(x)\cos(y) - \frac{\sqrt{3}}{4}x + \frac{\sqrt{3}}{4}y$
Use the Second Derivative Test to classify this critical point.

a. Local Maximum

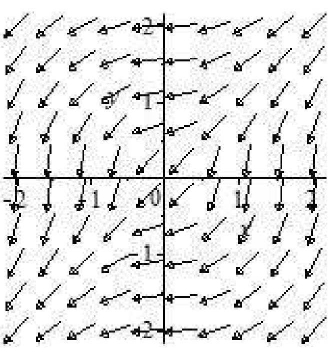
b. Local Minimum

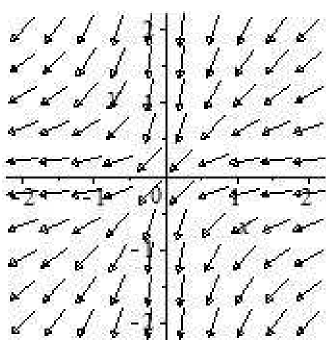
c. Inflection Point

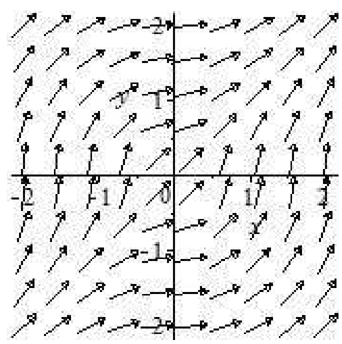
d. Saddle Point

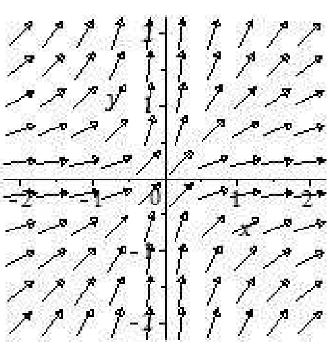
e. Test Fails

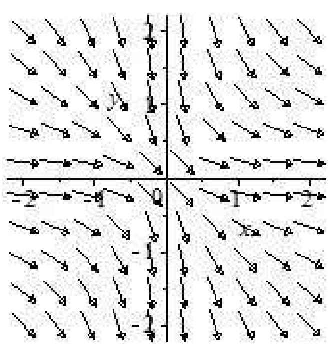
8. Which of the following is the plot of the vector field $\vec{F}(x,y) = \frac{1}{\sqrt{x^2+y^2}}\left(|x|,\ |y| \right)$?

a.


b.


c.


d.


e.


9. In $\mathbb{R}^4$, consider the parameric 2-surface $(x,y,z,w) = \vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, 2r\cos\theta, 2r\sin\theta)$ for $0 \le r \le 2$ and $0 \le \theta \le 2\pi$. Compute $\iint_{\vec{R}} (xz\,dy\,dz - y^2\,dz\,dw)$.

a. 32π
b. 16π
c. -16π
d. -32π
e. -64π

10. Find the equation of the plane tangent to the parameric surface $\vec{R}(u,t) = \left(ue^t, ue^{-t}, \sqrt{2}\,u\right)$ at the point $P = \vec{R}(2,0)$ where $u = 2$ and $t = 0$.
Hint: Evaluate the normal $\vec{N}$ at $u = 2$ and $t = 0$.

a. $x + y - \sqrt{2}\,z = -4\sqrt{2}$
b. $x + y - \sqrt{2}\,z = 0$
c. $x + y - \sqrt{2}\,z = 16\sqrt{2}$
d. $\sqrt{2}\,x - \sqrt{2}\,y + 2z = -4\sqrt{2}$
e. $\sqrt{2}\,x - \sqrt{2}\,y + 2z = 0$

11. If $\vec{F} = (xy\tan z, yz\cos x, xz\sin y)$, then $\vec{\nabla} \cdot \vec{\nabla} \times \vec{F} =$

a. $-2z\cos y - 2x(\tan^2 z + 1)$
b. $-2z\cos y + 2x(\tan^2 z + 1)$
c. $2z\cos y - 2x(\tan^2 z + 1)$
d. $2z\cos y + 2x(\tan^2 z + 1)$
e. 0

12. Let f be the scalar potential for $\vec{F} = (2xz - 3y, 8yz - 3x, x^2 + 4y^2 + 2z)$ for which $f(0,0,0) = 0$. Then $f(1,1,1) =$

a. 1
b. 2
c. 3
d. 4
e. 5

Work Out: (Points indicated. Part credit possible. Show all work.)

13. (15 points) The plane $x + 2y + 4z = 8$ intersects the 1st octant $(x > 0, y > 0, z > 0)$ in a triangle. Find the point on this triangle at which the function $f = xy^2z^3$ is a maximum.

14. (15 points) Compute $\iiint \vec{\nabla} \cdot \vec{F}\, dV$ for $\vec{F} = (xy^2, yz^2, zx^2)$ over the solid above the cone $z = \sqrt{x^2 + y^2}$ below the sphere $x^2 + y^2 + z^2 = 4$.

15. (15 points) Compute $\iint_E \vec{\nabla} \times \vec{F} \cdot \hat{k}\, dx\, dy$ for $\vec{F} = (-16x^2y, 9xy^2, 0)$ over the interior of the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

HINTS: First compute $\vec{\nabla} \times \vec{F} \cdot \hat{k}$ in rectangular coordinates.

Then compute the integral in elliptic coordinates $x = 3u\cos\theta$, $y = 4u\sin\theta$.