Consider the line \( X = P + tv \) where \( P = (2, 3, 2) \) and \( v = (2, -1, 2) \).

Drop a perpendicular from the point \( Q = (-1, 0, 5) \) to a point \( R \) on the line. Then \( R = P + \text{proj}_v PQ \).

\[ \text{proj}_v PQ = \frac{PQ \cdot v}{|v|^2} v = \frac{-6 + 3 + 6}{9} (2, -1, 2) = \left( \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right) \]

\[ R = P + \text{proj}_v PQ = \left( \frac{8}{3}, \frac{8}{3}, \frac{8}{3} \right) \]

If \( \vec{u} \) is 5 cm long and points 30° WEST of NORTH and \( \vec{v} \) is 4 cm long and points 30° EAST of NORTH, then \( \vec{u} \times \vec{v} \) is

a. 10 cm long and points DOWN.
b. 10 cm long and points UP.
c. 10 cm long and points SOUTH.
d. 10√3 cm long and points DOWN.   Correct Choice
e. 10√3 cm long and points SOUTH.

Since the angle between the vectors is \( \theta = 60° \), the length is

\[ |\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin \theta = 5 \cdot 4 \cdot \frac{\sqrt{3}}{2} = 10\sqrt{3} \]

Put your right hand fingers pointing 30° WEST of NORTH with the palm facing 30° EAST of NORTH, then your thumb points DOWN.

Find the point where the line \( (x, y, z) = (3 - 2t, 2 - t, 1 + t) \) intersects the plane \( x + y + 3z = 2 \). At this point, \( x + y + z = \)

a. 2
b. 4
c. 6
d. 8
e. The line does not intersect the plane.   Correct Choice

Substitute the line into the plane: \( (3 - 2t) + (2 - t) + 3(1 + t) = 2 \) or \( 8 = 2 \)

which is impossible. So the line does not intersect the plane.
4. The graph of the equation \( x^2 + 4x - y^2 + 4y + z^2 + 2z = -1 \) is a

a. hyperboloid of one sheet
b. hyperboloid of two sheets
c. cone Correct Choice
d. hyperbolic paraboloid
e. hyperbolic cylinder

\( x^2, y^2, \) and \( z^2 \) are all present with two +’s and one -. So this is a hyperboloid or cone.

Complete the squares to get \( (x + 2)^2 - (y - 2)^2 + (z + 1)^2 = 0 \) which is a cone.

5. For the helix \( \vec{r}(t) = (3t, \sin(4t), \cos(4t)) \), which of the following is FALSE?

a. \( \vec{v} = (3, 4 \cos(4t), -4 \sin(4t)) \)
b. \( \vec{a} = (0, -16 \sin(4t), -16 \cos(4t)) \)
c. \( \vec{j} = (0, -64 \cos(4t), 64 \sin(4t)) \)
d. speed = 25 Correct Choice
e. arclength between \((0, 0, 1)\) and \((3\pi, 0, 1)\) is \(5\pi\)

speed = \( \|\vec{v}\| = \sqrt{9 + 16 \cos^2(4t) + 16 \sin^2(4t)} = \sqrt{25} = 5 \)

6. For the helix \( \vec{r}(t) = (3t, \sin(4t), \cos(4t)) \), which of the following is FALSE?

a. \( \hat{T} = \left( \frac{3}{5}, \frac{4}{5} \cos(4t), -\frac{4}{5} \sin(4t) \right) \)
b. \( \hat{N} = (0, -\sin(4t), -\cos(4t)) \)
c. \( \hat{B} = \left( -\frac{4}{5}, -\frac{3}{5} \cos(4t), -\frac{3}{5} \sin(4t) \right) \) Correct Choice
d. \( a_T = 0 \)
e. \( a_N = 16 \)

\[
\vec{v} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 \cos(4t) & -4 \sin(4t) \\ 0 & -16 \sin(4t) & -16 \cos(4t) \end{vmatrix} = 16(-4, 3 \cos(4t), -3 \sin(4t))
\]

\[
|\vec{v} \times \vec{a}| = 16\sqrt{16 + 9 \cos^2(4t) + 9 \sin^2(4t)} = 80
\]

\[
\hat{B} = \frac{\vec{v} \times \vec{a}}{|\vec{v} \times \vec{a}|} = \left( -\frac{4}{5}, \frac{3}{5} \cos(4t), -\frac{3}{5} \sin(4t) \right)
\]
7. Which of the following is the contour plot of \( f(x,y) = y^2 + x + 1 \)?

- a.

- b.

- c.

- d.

- e. (a) Correct Choice

The contours are \( y^2 + x + 1 = C \) or \( x = C - 1 - y^2 \) which are parabolas opening left.

8. If \( P(2,3) = 5 \) and \( \frac{\partial P}{\partial x}(2,3) = 0.4 \) and \( \frac{\partial P}{\partial y}(2,3) = -0.3 \), estimate \( P(2.1,2.8) \).

- a. 4.9
- b. 4.98
- c. 4.99
- d. 5.01
- e. 5.1 Correct Choice

\[
P(x,y) = P(2,3) + P_x(2,3)(x-2) + P_y(2,3)(y-3)
\]

\[
P(2.1,2.8) = P(2,3) + P_x(2,3)(2.1-2) + P_y(2,3)(2.8-3)
\]

\[
= 5 + 0.4(1) - 0.3(-2) = 5.1
\]

9. Currently for a certain box, the length \( L \) is 5 cm and increasing at 0.2 cm/sec, the width \( W \) is 4 cm and decreasing at 0.3 cm/sec, the height \( H \) is 3 cm and increasing at 0.1 cm/sec. Then currently, the volume \( V \) is

- a. increasing at 0.1 cm/sec.
- b. decreasing at 0.1 cm/sec. Correct Choice
- c. increasing at 0.2 cm/sec.
- d. decreasing at 0.2 cm/sec.
- e. increasing at 0.3 cm/sec.

\[
V = LWH \quad \frac{dL}{dt} = 0.2 \quad \frac{dW}{dt} = -0.3 \quad \frac{dH}{dt} = 0.1
\]

\[
\frac{dV}{dt} = \frac{\partial V}{\partial L} \frac{dL}{dt} + \frac{\partial V}{\partial W} \frac{dW}{dt} + \frac{\partial V}{\partial H} \frac{dH}{dt} = WH \frac{dL}{dt} + LH \frac{dW}{dt} + LW \frac{dH}{dt}
\]

\[
= 4 \cdot 3 \cdot 0.2 - 5 \cdot 3 \cdot 0.3 + 5 \cdot 4 \cdot 0.1 = -0.1
\]
10. The temperature of a frying pan is \( T = \frac{1}{1 + x^2 + 4y^2} \). An ant is located at (2, 1). In what **unit vector** direction should the ant move to **decrease** the temperature as fast as possible?

a. \((-1, -2)\)  

b. \((1, 2)\) Part Credit  

c. \((1, -2)\)  

d. \(\left(\frac{-1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}\right)\) Part Credit  

e. \(\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)\) Correct Choice

\[
\nabla T = \left(\frac{\frac{-2x}{(1 + x^2 + 4y^2)^2}, \frac{-8y}{(1 + x^2 + 4y^2)^2}}{81} \right) \quad \nabla T\big|_{(2,1)} = \left(\frac{-4}{81}, \frac{-8}{81}\right)
\]

\[
|\nabla T| = \frac{\sqrt{16 + 64}}{81} = \frac{\sqrt{80}}{81} = \frac{4\sqrt{5}}{81}
\]

Maximum decrease is in the direction of \( \vec{v} = -\nabla T\big|_{(2,1)} = \left(\frac{-4}{81}, \frac{-8}{81}\right) \).

The unit vector is \( \frac{\vec{v}}{|\vec{v}|} = \frac{1}{4\sqrt{5}} \left(\frac{4}{81}, \frac{8}{81}\right) = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) \).

11. The temperature of a frying pan is \( T = \frac{1}{1 + x^2 + 4y^2} \). An ant is located at (2, 1) and has velocity \( \vec{v} = (0.3, -0.6) \). What is the rate of change of the temperature as seen by the ant?

a. \(-0.444\)  

b. \(-0.2\)  

c. \(0.2\)  

d. \(0.3333\)  

e. \(0.0444\) Correct Choice

\[
\frac{dT}{dt} = \vec{v} \cdot \nabla T\big|_{(2,1)} = (0.3, -0.6) \cdot \left(\frac{-4}{81}, \frac{-8}{81}\right) = \frac{-1.2 + 4.8}{81} = \frac{0.4}{9} = \frac{2}{45} \approx 0.0444
\]

12. The point (2, 1, -1) is on the graph of \( x^2yz^2 + xy^3z = 2 \). Compute \( \frac{\partial z}{\partial y}\big|_{(2,1)} \).

a. \(-\frac{2}{3}\)  

b. \(-\frac{1}{3}\) Correct Choice  

c. \(\frac{1}{3}\)  

d. \(\frac{2}{3}\)  

e. \(\frac{4}{3}\)

Apply \( \frac{\partial}{\partial y} \) to both sides and evaluate at (2, 1, -1).

\[
x^2z^2 + 2x^2yz\frac{\partial z}{\partial y} + 3xy^2z + xy^3\frac{\partial z}{\partial y} = 0 \quad 4 - 8 \frac{\partial z}{\partial y} - 6 + 2 \frac{\partial z}{\partial y} = 0 \quad \frac{\partial z}{\partial y} = -\frac{1}{3}
\]
13. (11 points) Find the mass of the helical wire \( \vec{r}(t) = (3t, \sin(4t), \cos(4t)) \) from \((0, 0, 1)\) to \((3\pi, 0, 1)\) if its linear density is \( \rho = x^2 + y^2 + z^2 \).

\[
\vec{v} = (3, 4 \cos(4t), -4 \sin(4t)) \quad [\vec{v}] = 9 + 16 \cos^2(4t) + 16 \sin^2(4t) = 5
\]
\[
\rho(\vec{r}(t)) = 9t^2 + \sin^2(4t) + \cos^2(4t) = 9t^2 + 1
\]
\[
M = \int_{(0,0,1)}^{(3\pi,0,1)} \rho \, ds = \int_0^\pi \rho(\vec{r}(t))|\vec{v}| \, dt = \int_0^\pi (9t^2 + 1) \, dt = 5[3t^3 + t]_0^\pi = 5(3\pi^3 + \pi)
\]

14. (11 points) A bead slides along the helix \( \vec{r}(t) = (3t, \sin(4t), \cos(4t)) \) from \((0, 0, 1)\) to \((3\pi, 0, 1)\) under the action of the force \( \vec{F} = (x, xy, xz) \). Find the work done.

\[
\vec{v} = (3, 4 \cos(4t), -4 \sin(4t)) \quad \vec{F}(\vec{r}(t)) = (3t, 3t \sin(4t), 3t \cos(4t))
\]
\[
\vec{F} \cdot \vec{v} = 9t + 12t \cos(4t) \sin(4t) - 12t \sin(4t) \cos(4t) = 9t
\]
\[
W = \int_{(0,0,1)}^{(3\pi,0,1)} \vec{F} \cdot d\vec{s} = \int_0^\pi \vec{F} \cdot \vec{v} \, dt = \int_0^\pi 9t \, dt = \left[ \frac{9t^2}{2} \right]_0^\pi = \frac{9}{2} \pi^2
\]

15. (11 points) Find the plane tangent to the graph of the function \( z = x^2y + y^3x \) at the point \((x, y) = (2, 1)\). Find the \( z \)-intercept.

\[
f(x, y) = x^2y + y^3x, \quad f_x(x, y) = 2xy + y^3, \quad f_y(x, y) = x^2 + 3y^2x, \quad f(2, 1) = 6, \quad f_x(2, 1) = 5, \quad f_y(2, 1) = 10
\]
\[
z = f(2, 1) + f_x(2, 1)(x - 2) + f_y(2, 1)(y - 1) = 6 + 5(x - 2) + 10(y - 1) = 5x + 10y - 14
\]

The plane is \( z = 5x + 10y - 14 \). The \( z \)-intercept is \( -14 \).

16. (12 points) Find the plane tangent to the level surface \( x \sin z + y \cos z = 3 \) at the point \((x, y, z) = (3, 2, \frac{\pi}{2})\). Find the \( z \)-intercept.

\[
F(x, y, z) = x \sin z + y \cos z \quad \vec{\nabla}F = (\sin z, \cos z, x \cos z - y \sin z) \quad \vec{N} = \left. \vec{\nabla}F \right|_{(3,2,\pi/2)} = (1, 0, -2)
\]
\[
\vec{N} \cdot \vec{X} = \vec{N} \cdot P \quad x - 2z = 3 - 2\left( \frac{\pi}{2} \right) = 3 - \pi
\]

The plane is \( x - 2z = 3 - \pi \) or \( z = \frac{1}{2}(x + \pi - 3) = \frac{1}{2}x + \frac{\pi}{2} - \frac{3}{2} \). The \( z \)-intercept is \( \frac{\pi}{2} - \frac{3}{2} \).