Name______________________ Sec________

MATH 251 Final Exam Spring 2011

Section 200/511 P. Yasskin

1-12	/48	14	/16
13	/16	15	/25
		Total	/105

Multiple Choice: (4 points each. No part credit.)

1. Find the projection of the vector $\vec{u} = (12,-3,4)$ along the vector $\vec{v} = (2,1,-2)$.

a. $\left(\frac{-12}{13}, \frac{3}{13}, \frac{-4}{13}\right)$

b. $\left(\frac{12}{13}, \frac{-3}{13}, \frac{4}{13}\right)$

c. $\left(\frac{-26}{9}, \frac{-13}{9}, \frac{26}{9}\right)$

d. $\left(\frac{26}{9}, \frac{13}{9}, \frac{-26}{9}\right)$

e. $\left(\frac{108}{13}, \frac{-9}{13}, \frac{36}{13}\right)$

2. If $|\vec{u}| = 4$ and $|\vec{v}| = 3$ and the angle between $\vec{u}$ and $\vec{v}$ is $\theta = \frac{\pi}{3}$ then $|\vec{u} \times \vec{v}| =$

a. $6\sqrt{3}$

b. 6

c. $3\sqrt{3}$

d. 3

e. $12\sqrt{3}$

3. Find the point where the line through the origin perpendicular to the plane $3x - 3y + 4z = 17$ intersects that plane. At this point $x + y + z =$

a. 2

b. 3

c. 5

d. 17

e. $\frac{1}{2}$

4. Find the arc length of the curve $\vec{r}(t) = (t^2, 2t, \ln t)$ between $(1,2,0)$ and $(e^2, 2e, 1)$.

a. e^2
b. $e^2 + 1$
c. $e^2 - 1$
d. $\frac{2}{3}e^3 + e$
e. $\frac{2}{3}e^3 + e - \frac{5}{3}$

5. The pressure, P, density, ρ, and temperature, T, of a certain ideal gas are related by $P = 3\rho T$.
At the point $(1,2,3)$, the density and its gradient are $\rho = 4$ and $\vec{\nabla}\rho = (0.2, 0.4, 0.1)$,
while the temperature and its gradient are $T = 300$ and $\vec{\nabla}T = (2,1,3)$.
Hence the pressure is $P = 3 \cdot 4 \cdot 300 = 3600$ and its gradient is $\vec{\nabla}P =$

a. $(1802.4, 2704.8, 2701.2)$
b. $(600.8, 901.900.4)$
c. $(68, 132, 42)$
d. $(204, 372, 126)$
e. $(1.2, 1.2, 0.9)$

6. Find the equation of the plane tangent to the graph of $x^2yz^2 - 2y^2z^3 = 10$ at the point $(3,2,1)$.
The z-intercept is

a. $\frac{6}{25}$
b. 5
c. $\frac{25}{6}$
d. 60
e. $\frac{5}{3}$

7. Han Deut is flying the Millennium Eagle through a dangerous zenithon field whose density is $\rho = xyz$. If his current position is $(x,y,z) = (1,-1,2)$, in what **unit** vector direction should he travel to **decrease** the density as fast as possible?

a. $(2,-2,1)$

b. $\left(\frac{2}{3},\frac{-2}{3},\frac{1}{3}\right)$

c. $(-2,2,-1)$

d. $\left(\frac{-2}{3},\frac{2}{3},\frac{-1}{3}\right)$

e. $\left(\frac{-2}{3},\frac{-2}{3},\frac{-1}{3}\right)$

8. Compute $\iint e^{-x^2-y^2}\,dA$ over the quarter circle $x^2+y^2 \le 9$ in the first quadrant.

a. $\frac{\pi}{2}e^{-9}$

b. $\frac{\pi}{2}(e^{-9}-1)$

c. $-\frac{\pi}{4}e^{-9}$

d. $\frac{\pi}{4}(1-e^{-9})$

e. $\pi(1-e^{-9})$

9. Compute $\int_0^8 \int_{x^{1/3}}^2 \cos(y^4)\,dy\,dx$

HINT: Reverse the order of integration.

a. $\frac{1}{4}\sin(4)-\frac{1}{4}$

b. $\frac{1}{4}\sin(64)-\frac{1}{4}$

c. $\frac{1}{4}\sin(64)$

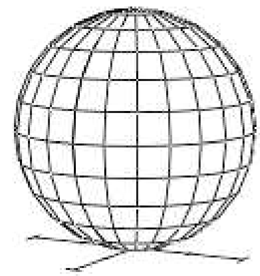
d. $\frac{1}{4}\sin(16)-\frac{1}{4}$

e. $\frac{1}{4}\sin(16)$

10. Compute $\iiint z\,dV$ over the solid sphere $x^2 + y^2 + (z-1)^2 \le 1$

given in spherical coordinates by $\rho = 2\cos\varphi$.

HINT: The whole sphere has $z \ge 0$. What does this say about φ?



a. $\frac{2\pi}{3}$

b. $\frac{4\pi}{3}$

c. $\frac{4\pi}{6}$

d. $\frac{\pi}{3}$

e. $\frac{\pi}{12}$

11. Compute $\int \vec{F} \cdot d\vec{s}$ for $\vec{F} = (2x, 2y, 2z)$ along the curve $\vec{r}(t) = \left(\frac{2}{t}, \frac{4}{t}, \frac{6}{t}\right)$

from $(2,4,6)$ to $(1,2,3)$.

HINT: Find a scalar potential.

a. -70

b. -42

c. 0

d. 42

e. 70

12. Compute $\oint \vec{F} \cdot d\vec{s}$ for $\vec{F} = (\sec(x^3) - 5y, \cos(y^5) + 3x)$ counterclockwise around the triangle with vertices $(0,0)$, $(8,0)$ and $(0,4)$.

Hint: Use Green's Theorem.

a. 12

b. 16

c. 32

d. 64

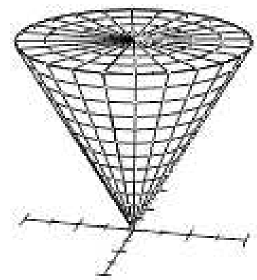
e. 128

Work Out: (Points indicated. Part credit possible. Show all work.)

13. (16 points) Use Lagrange multipliers to find 4 numbers, a, b, c, and d, whose product is $\frac{2}{3}$ and for which $a + 2b + 3c + 4d$ is a minimum.

14. (16 points) Find the mass and the y-component of the center of mass of the quarter of the cylinder $x^2 + y^2 \leq 4$ with $0 \leq z \leq 3$ in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ if the mass density is $\delta = xyz$.

15. (25 points) Verify Gauss' Theorem $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV = \iint\limits_{\partial V} \vec{F} \cdot d\vec{S}$



for the vector field $\vec{F} = (2xy^2, 2yx^2, z(x^2+y^2))$ and the solid above the cone $z = 2\sqrt{x^2+y^2}$ below the plane $z = 2$.

Be careful with orientations. Use the following steps:

First the Left Hand Side:

a. Compute the divergence:

$\vec{\nabla} \cdot \vec{F} =$

b. Express the divergence and the volume element in the appropriate coordinate system:

$\vec{\nabla} \cdot \vec{F} =$ $dV =$

c. Find the limits of integration:

d. Compute the left hand side:

$\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV =$

Second the Right Hand Side:

The boundary surface consists of a cone C and a disk D with appropriate orientations.

e. Complete the parametrization of the cone C:

$\vec{R}(r,\theta) = \left(r\cos\theta, r\sin\theta, \underline{\qquad\qquad}\right)$

f. Compute the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

g. Compute the normal vector:

$\vec{N} =$

h. Evaluate $\vec{F} = (2xy^2, 2yx^2, z(x^2+y^2))$ on the cone:

$$\vec{F}\Big|_{\vec{R}(r,\theta)} =$$

i. Compute the dot product:

$$\vec{F} \bullet \vec{N} =$$

j. Compute the flux through C:

$$\iint_C \vec{F} \bullet d\vec{S} =$$

k. Complete the parametrization of the disk D:

$$\vec{R}(r,\theta) = \left(r\cos\theta, r\sin\theta, \underline{\qquad\qquad}\right)$$

l. Compute the tangent vectors:

$$\vec{e}_r =$$

$$\vec{e}_\theta =$$

m. Compute the normal vector:

$$\vec{N} =$$

n. Evaluate $\vec{F} = (2xy^2, 2yx^2, z(x^2+y^2))$ on the disk:

$$\vec{F}\Big|_{\vec{R}(r,\theta)} =$$

o. Compute the dot product:

$$\vec{F} \bullet \vec{N} =$$

p. Compute the flux through D:

$$\iint_D \vec{F} \bullet d\vec{S} =$$

q. Compute the right hand side:

$$\iint_{\partial V} \vec{F} \bullet d\vec{S} =$$