Name________________________ ID__________________

1-12	/60
13	/10
14	/10
15	/10
16	/10
Total	/100

MATH 251 Exam 1 Version A Spring 2013

Sections 506 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Find the line through $P = (1,2,3)$ which is perpendicular to both of the vectors $\vec{a} = \langle 3,-1,2\rangle$ and $\vec{b} = \langle 1,0,-2\rangle$.

a. $(x,y,z) = (2+t, 8+2t, 1+3t)$

b. $(x,y,z) = (2+t, -8+2t, 1+3t)$

c. $(x,y,z) = (2-t, -8-2t, 1-3t)$

d. $(x,y,z) = (1+2t, 2-8t, 3+t)$

e. $(x,y,z) = (1+2t, 2+8t, 3+t)$

2. A triangle has vertices at $A = \langle 1,1,1\rangle$, $B = \langle 3,4,-3\rangle$ and $C = \langle 3,3,2\rangle$.
Drop a perpendicular from B to the side $\overline{AC}$.
Find the point P where the perpendicular intersects the side $\overline{AC}$.

a. $\left\langle \frac{7}{3}, \frac{7}{3}, \frac{5}{3} \right\rangle$

b. $\left\langle \frac{4}{3}, \frac{4}{3}, \frac{2}{3} \right\rangle$

c. $\left\langle \frac{5}{3}, \frac{5}{3}, \frac{4}{3} \right\rangle$

d. $\left\langle \frac{41}{29}, \frac{47}{29}, \frac{5}{29} \right\rangle$

e. $\left\langle \frac{12}{29}, \frac{18}{29}, \frac{-24}{29} \right\rangle$

3. If $\vec{u}$ points NorthEast and $\vec{v}$ points Down, then $\vec{u} \times \vec{v}$ points

a. NorthEast

b. NorthWest

c. SouthEast

d. SouthWest

e. Up

4. Identify the quadratic surface for the equation

$$2(x-2)^2 + (y-3)^2 + (z-2)^2 = (x-2)^2 + 2(y-3)^2 + (z+2)^2$$

a. hyperboloid of 1 sheet

b. hyperboloid of 2 sheets

c. cone

d. hyperbolic paraboloid

e. hyperbolic cylinder

5. A girl scout is hiking up a mountain whose attitude is given by $z = h(x,y) = 10 - x - x^2 - y^2$. If she is currently at the point $(x,y) = (1,2)$, in what unit vector direction should she walk to go up hill as fast as possible?

a. $\left(-\frac{3}{5}, -\frac{4}{5}\right)$

b. $\left(\frac{3}{5}, \frac{4}{5}\right)$

c. $\left(-\frac{4}{5}, -\frac{3}{5}\right)$

d. $\left(\frac{4}{5}, \frac{3}{5}\right)$

e. $(4,3)$

6. Find the arclength of 4 revolutions around the helix $\vec{r}(t) = (2\cos 2t, \quad 2\sin 2t, \quad 3t)$.
NOTE: Each revolution covers an angle of 2π. How much does t change?

a. 2π

b. 4π

c. 5π

d. 15π

e. 20π

7. A wire in the shape of the helix $\vec{r}(t) = (2\cos 2t, \quad 2\sin 2t, \quad 3t)$ has linear mass density $\rho = z^2$.
Find its total mass between $t = 0$ and $t = 2\pi$.

a. $M = 24\pi^3$

b. $M = 36\pi^2$

c. $M = 120\pi^3$

d. $M = 180\pi^2$

e. $M = 240\pi^2$

8. Find the work done to move an object along the helix $\vec{r}(t) = (2\cos 2t, \quad 2\sin 2t, \quad 3t)$
between $t = 0$ and $t = 2\pi$ by the force $\vec{F} = \langle -yz, \quad xz, \quad z\rangle$.

a. $\frac{33}{2}\pi$

b. 33π

c. $\frac{33}{2}\pi^2$

d. $33\pi^2$

e. $66\pi^2$

9. Find the tangent line to the helix $\vec{r}(t) = (2\cos 2t, \quad 2\sin 2t, \quad 3t)$ at the point $t = \frac{\pi}{2}$.
Where does it intersect the xy-plane?
HINT : What are the position and tangent vector at $t = \frac{\pi}{2}$?

a. $(x,y) = (-1,\pi)$

b. $(x,y) = (-1,-\pi)$

c. $(x,y) = (-2,2\pi)$

d. $(x,y) = (-2,-2\pi)$

e. $(x,y) = (2,\pi)$

10. Find the plane tangent to the graph of $z = y\ln x$ at the point $(e,2)$. Its z-intercept is

a. $-e$

b. -2

c. 0

d. 2

e. e

11. Find the plane tangent to the graph of $x^2z^2 + 2zy^2 + yx^3 = 71$ at the point $(2, 1, 0)$. Its z-intercept is

a. 2

b. 4

c. 8

d. 16

e. 32

12. The point $(x, y) = \left(1, \frac{1}{2}\right)$ is a critical point of the function $f(x, y) = 4xy - x^3y - 4xy^3$. Use the Second Derivative Test to classify this critical point.

a. local minimum

b. local maximum

c. saddle point

d. TEST FAILS

Work Out: (10 points each. Part credit possible. Show all work.)

13. Find the point where the line $(x, y, z) = (4 + 3t, 3 - 2t, 2 + t)$ intersects the plane $x + 2y + 3z = 20$, or explain why they are parallel.

14. Find the line where the planes $-2x - 6y + 4z = 7$ and $3x + 9y - 6z = 5$ intersect, or explain why they are parallel.

15. If two adjustable resistors, with resistances R_1 and R_2, are arranged in parallel, the total resistance R is given by

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

Currently, $R_1 = 3\Omega$ and $R_2 = 7\Omega$ and they are changing according to $\frac{dR_1}{dt} = -0.1\ \frac{\Omega}{\text{sec}}$ and $\frac{dR_2}{dt} = 0.2\ \frac{\Omega}{\text{sec}}$. Find R and $\frac{dR}{dt}$. Is R increasing or decreasing?

16. A rectangular box sits on the xy-plane with its top 4 vertices in the paraboloid $z = 8 - 2x^2 - 8y^2$. Find the dimensions and volume of the largest such box.