1. Compute $\int \int_{R} xy \, dA$ over the region $R$ between the parabola $y = x^2$ and the line $y = 2x$.

   a. $\frac{29}{6}$
   b. $\frac{10}{3}$
   c. $\frac{32}{15}$
   d. $\frac{8}{3}$
   e. $\frac{4}{3}$

2. Compute $\int_{0}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} x \, dy \, dx$ by converting to polar coordinates.

   a. 3
   b. 9
   c. 18
   d. 27
   e. 36
3. Find the mass of the triangular plate with vertices \((0,0), (2, -6)\) and \((2, 6)\) if the surface density is \(\rho = x^2\).
   a. 24
   b. 16
   c. 12
   d. 6
   e. 4

4. Find the center of mass of the triangular plate with vertices \((0,0), (2, -6)\) and \((2, 6)\) if the surface density is \(\rho = x^2\).
   a. \(\left(\frac{6}{5}, 0\right)\)
   b. \(\left(\frac{5}{9}, 0\right)\)
   c. \(\left(\frac{5}{8}, 0\right)\)
   d. \(\left(\frac{9}{5}, 0\right)\)
   e. \(\left(\frac{8}{5}, 0\right)\)

5. Which of the following integrals is NOT equivalent to \(\int_0^4 \int_0^{y^2} \int_0^z f(x, y, z) \, dx \, dy \, dz?\) The region is shown.
   a. \(\int_0^4 \int_0^{\sqrt{z}} \int_0^{\sqrt{z}} f(x, y, z) \, dx \, dy \, dz\)
   b. \(\int_0^4 \int_0^{\sqrt{z}} f(x, y, z) \, dz \, dx \, dy\)
   c. \(\int_0^4 \int_0^{y^2} \int_0^z f(x, y, z) \, dx \, dz \, dy\)
   d. \(\int_0^2 \int_0^4 \int_0^{z} f(x, y, z) \, dz \, dy \, dx\)
   e. \(\int_0^2 \int_0^{4-x^2} \int_0^{z} f(x, y, z) \, dy \, dz \, dx\)
6. Find the area of one petal of the 8 petaled daisy \( r = \sin(4\theta) \).
   
   a. \( \frac{\pi}{32} \)
   b. \( \frac{\pi}{16} \)
   c. \( \frac{\pi}{8} \)
   d. \( \frac{\pi}{4} \)
   e. \( \frac{\pi}{2} \)

7. Find the mass of the solid between the paraboloids \( z = x^2 + y^2 \) and \( z = 8 - x^2 - y^2 \) if the volume density is \( \rho = z \).
   
   a. \( 4\pi \)
   b. \( 8\pi \)
   c. \( 16\pi \)
   d. \( 32\pi \)
   e. \( 64\pi \)

8. Compute \( \iiint (x^2 + y^2) \, dz \, dV \) over the solid hemisphere \( 0 \leq \sqrt{x^2 + y^2 + z^2} \leq 2 \)
   
   a. \( \frac{8}{3} \pi \)
   b. \( \frac{16}{3} \pi \)
   c. \( \frac{64}{9} \pi \)
   d. \( 2\pi \)
   e. \( 0 \)
9. Which integral gives the arc length of the ellipse \( \vec{r}(\theta) = (6 \cos \theta, 3 \sin \theta, 3 \sin \theta) \)?

a. \( \int_0^{2\pi} \sqrt{54} \, d\theta \)

b. \( \int_0^{2\pi} 3\sqrt{2 + 2 \sin^2 \theta} \, d\theta \)

c. \( \int_0^{2\pi} 3\sqrt{2 + 2 \cos^2 \theta} \, d\theta \)

d. \( \int_0^{2\pi} 3\sqrt{4 + 2 \cos^2 \theta} \, d\theta \)

e. \( \int_0^{2\pi} \sqrt{3} (3 + 3 \sin^2 \theta) \, d\theta \)

10. A helical thermocouple whose shape is the curve \( \vec{r}(t) = (3 \cos t, 3 \sin t, 4t) \) for \( 0 \leq t \leq 4\pi \) is placed in a pot of water where the temperature is \( T = (41 + x^2 + y^2 + z)^\circ \text{C} \). Find the average temperature of the water as measured by the thermocouple.

HINT: \( f_{\text{ave}} = \frac{\int_{\alpha}^{\beta} f(\alpha, \beta) \, ds}{\int_{\alpha}^{\beta} ds} \)

a. 50 + 8\pi

b. 50 + 16\pi

c. 1000\pi + 160\pi^2

d. 250 + 40\pi

e. \frac{173}{4} + 8\pi
11. Find a scalar potential, \( f(x, y, z) \), for \( \vec{F}(x, y, z) = (2xy^2 + 2x + 2xz, 2x^2y - 3z, x^2 + 3z^2 - 3y) \). Then compute \( f(2, 2, 2) - f(1, 1, 1) \).

a. 25  
b. 23  
c. 7  
d. 1 
e. 0

12. If \( f = x^2 + y^2 - 2z^2 \) and \( \vec{F} = (xz, yz, -z^2) \), which of the following is false?

a. \( \nabla \cdot \nabla f = 0 \)  
b. \( \nabla \times \nabla f = 0 \)  
c. \( \nabla \left( \nabla \cdot \vec{F} \right) = 0 \)  
d. \( \nabla \cdot \nabla \times \vec{F} = 0 \)  
e. None of the above. They are all true.

13. Compute \( \iiint_C \nabla \cdot \vec{G} \, dV \) for \( \vec{G} = (xz, yz, z^2) \) over the solid cylinder \( x^2 + y^2 \leq 25 \) with \( 0 \leq z \leq 4 \).

a. \( 40\pi \)  
b. \( 80\pi \)  
c. \( 200\pi \)  
d. \( 400\pi \)  
e. \( 800\pi \)
14. (12 points) Compute $\iint_D x^2 dA$ over the "diamond" shaped region bounded by the curves
$y = 1 + x^3$, $y = 3 + x^3$, $y = 4 - x^3$, $y = 7 - x^3$.
HINT: Define curvilinear coordinates $(u, v)$ so that
$y = u + x^3$ and $y = v - x^3$.

a. (2 pts) What are the boundaries in terms of $u$ and $v$?

b. (3 pts) Find formulas for $x$ and $y$ in terms of $u$ and $v$.

c. (4 pts) Find the Jacobian factor $J = \left| \frac{\partial(x, y)}{\partial(u, v)} \right|$.

d. (1 pts) Express the integrand in terms of $u$ and $v$.

e. (2 pts) Compute the integral.
Consider the elliptical region, $E$, in the plane $z = 2 + x + y$ above the circle $x^2 + y^2 \leq 4$ oriented upwards.

**HINT:** This ellipse may be parametrized by $\vec{R}(r, \theta) = (r \cos \theta, r \sin \theta, 2 + r \cos \theta + r \sin \theta)$.

**a. (10 pts)** Find the normal vector $\vec{N}$ to the ellipse and its length $|\vec{N}|$.

Note: $\vec{N}$ starts hard but simplifies!

**b. (3 pts)** Find the surface area of the ellipse.

**c. (3 pts)** Find the mass of the ellipse if the surface density is $\rho = x^2 + y^2$.

**d. (12 pts)** If $\vec{F} = (-yz, xz, z^2)$, compute the surface integral $\iiint_E \vec{N} \times \vec{F} \cdot d\vec{S}$.
16. (12 points) Compute \( \int_C \vec{G} \cdot d\vec{S} \) for \( \vec{G} = (xz, yz, z^2) \) over the cylinder \( x^2 + y^2 = 25 \) for \( 0 \leq z \leq 4 \) with outward normal.