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MATH 251 Final Exam Spring 2013

Sections 506 P. Yasskin

Multiple Choice: (4 points each. No part credit.)

1. How much work is done to push a box up an incline from $(2,1)$ to $(4,2)$ by the force $\vec{F} = (3,2)$?

a. 65

b. $\sqrt{65}$

c. $2\sqrt{65}$

d. 64

e. 8

2. The graph of $2x^2 + 4x + 3y - 3z^2 - 6z = 4$ is a

a. hyperboloid of 1-sheet

b. hyperboloid of 2-sheets

c. elliptic paraboloid

d. hyperbolic paraboloid

e. cone

3. Find the tangential acceleration, a_T, or the curve $\vec{r}(t) = \left(t^2, \frac{4}{3}t^3, t^4\right)$. HINT: Perfect square.

a. $2 - 12t^2$

b. $2 + 12t^2$

c. $2t - 4t^3$

d. $2t + 4t^3$

e. $2 + 16t^2$

4. Find the equation of the plane through the points $(1,2,3)$, $(2,4,2)$ and $(-1,2,4)$.
Its z-intercept is

a. 1
b. 2
c. 4
d. 8
e. 16

5. Find the plane tangent to the graph of the function $z = 2x^2y$ at the point $(x,y) = (3,2)$.
Its z-intercept is

a. -72
b. -36
c. 0
d. 36
e. 72

6. Find the plane tangent to the level set of the function $F(x,y,z) = 2x^2yz^3$ at the point $(x,y,z) = (3,2,1)$. Its z-intercept is

a. 1
b. 2
c. 3
d. 108
e. 216

7. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.
If the radius r is currently 3 cm and decreasing at 2 cm/sec
while the height h is currently 4 cm and increasing at 1 cm/sec,
is the volume increasing or decreasing and at what rate?

a. increasing at 19π cm$^3/\text{sec}$

b. increasing at 13π cm$^3/\text{sec}$

c. neither increasing nor decreasing

d. decreasing at 13π cm$^3/\text{sec}$

e. decreasing at 19π cm$^3/\text{sec}$

8. Duke Skywater is traveling in the Millenium Eagle through a dangerous galactic politon field whose density is $\rho = 2x^2yz^3$. If Duke's current position and velocity are $\vec{r} = (3,2,1)$ and $\vec{v} = (.25,.5,-.25)$, what is the current time rate of change of the politon field as seen by Duke?

a. -12

b. -120

c. 12

d. 120

e. $12,564$

9. Duke Skywater is traveling in the Millenium Eagle through a dangerous galactic politon field whose density is $\rho = 2x^2yz^3$. If Duke's current position is $\vec{r} = (3,2,1)$, in what unit vector direction should he travel to **reduce** the politon field as fast as possible?

a. $\frac{1}{\sqrt{349}}(4,3,18)$

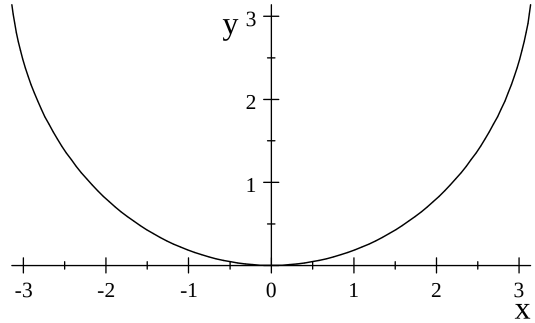
b. $\frac{1}{\sqrt{349}}(4,-3,18)$

c. $\frac{1}{\sqrt{349}}(-4,-3,-18)$

d. $\frac{1}{\sqrt{349}}(-4,3,-18)$

10. Compute $\int \vec{F} \cdot d\vec{s}$ where $\vec{F} = (2x + 2y, 2x + 2y)$ along the curve $\vec{r}(t) = \left(4\sqrt{2}\, t\cos(t), 4\sqrt{2}\, t\sin(t)\right)$ for $-\frac{\pi}{4} \le t \le \frac{\pi}{4}$.

HINT: Find a scalar potential.



a. 2π

b. 4π

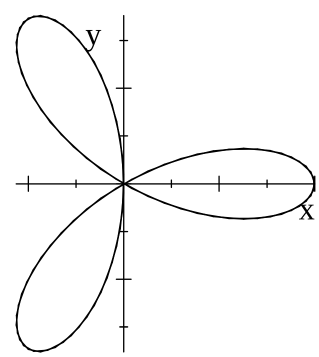
c. 8π

d. $4\pi^2$

e. $8\pi^2$

11. Compute $\oint \vec{F} \cdot d\vec{s}$ for $\vec{F} = (x - y, x + y)$ counterclockwise around the one leaf of the 3 leaf rose $r = \cos(3\theta)$ with $x \ge 0$.

HINT: Use Green's Theorem.



a. $\frac{\pi}{2}$

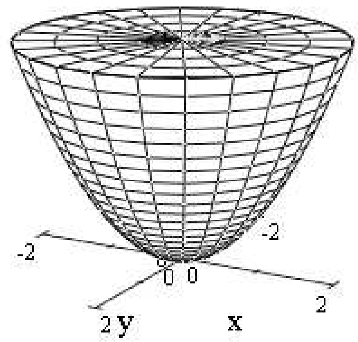
b. $\frac{\pi}{3}$

c. $\frac{\pi}{4}$

d. $\frac{\pi}{6}$

e. $\frac{\pi}{12}$

12. Compute $\iint \vec{F} \cdot d\vec{S}$ for $\vec{F} = (xy^2, yx^2, z(x^2 + y^2))$ over the complete surface of the solid above the paraboloid $z = x^2 + y^2$ below the plane $z = 4$, oriented outward.



a. $\frac{64}{5}\pi$

b. $\frac{64}{3}\pi$

c. $\frac{64}{15}\pi$

d. $\frac{256}{15}\pi$

e. $\frac{256}{3}\pi$

13. Compute $\iint (x^3 + xy^2)\, dx\, dy$ over the quarter circle $x^2 + y^2 \le 4$ in the first quadrant.

a. 4

b. $\frac{8}{5}\pi$

c. $\frac{16}{5}\pi$

d. $\frac{64}{5}\pi$

e. $\frac{32}{5}$

Work Out: (Points indicated. Part credit possible. Show all work.)

14. (15 points) The half cylinder $x^2 + y^2 = 9$ for $y \geq 0$ and $0 \leq z \leq 4$ has mass surface density $\rho = y^2$. Find the total mass and the center of mass. Follow these steps:

Parametrize the surface:

$\vec{R}(\quad , \quad) = (\qquad , \qquad , \qquad)$

Find the tangent vectors, the normal vector and its length:

$\vec{e}_\theta =$

$\vec{e}_z =$

$\vec{N} =$

$\left|\vec{N}\right| =$

Evaluate the density on the surface and compute the total mass:

$\rho = y^2 \qquad \rho|_{\vec{R}} =$

$M =$

Use symmetry to determine the x and z components fo the center of mass and then compute the y component of the center of mass.

$\bar{x} = \qquad\qquad \bar{z} =$

$M_{xz} =$

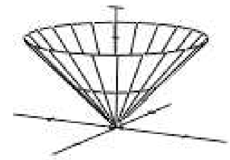
$\bar{y} =$

15. (15 points) A rectangular box sits on the xy-plane with its top 4 vertices on the paraboloid $z + 2x^2 + 8y^2 = 8$. Find the dimensions and volume of the largest such box.

NOTE: Full Credit for solving by Lagrange Multipliers, Half Credit for Eliminating a Variable.

16. (20 points) Verify Stokes' Theorem $\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial C} \vec{F} \cdot d\vec{s}$

for the cone C given by $z = \sqrt{x^2 + y^2}$ for $z \le 3$



oriented down and out, and the vector field $\vec{F} = (-yz, xz, z^2)$.

Note: The boundary of the cone is the circle, $x^2 + y^2 = 9$,

Be sure to check the orientations. Use the following steps:

a. The cone, C, may be parametrized as $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$

Compute the surface integral by successively finding:

$\vec{e}_r$, $\vec{e}_\theta$, $\vec{N}$, $\vec{\nabla} \times \vec{F}$, $\vec{\nabla} \times \vec{F}\big|_{\vec{R}(r,\theta)}$, $\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S}$

b. Parametrize the circle, ∂C, and compute the line integral by successively finding:

$\vec{r}(\theta)$, $\vec{v}$, $\vec{F}\big|_{\vec{r}(\theta)}$, $\int_{\partial C} \vec{F} \cdot d\vec{s}$