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MATH 251 Take Home Quiz 1 Spring 2013
Sections 506 P. Yasskin

Prove the Pythagorean Identity for Vectors:
\[(\vec{u} \cdot \vec{v})^2 + |\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2\]
as follows: Consider the vectors \( \vec{u} = (u_1, u_2, u_3) \) and \( \vec{v} = (v_1, v_2, v_3) \).
Compute each of the following by hand on paper. Show your work. Simplify where possible.

1. Write the algebraic definition for \( \vec{u} \cdot \vec{v} \).
\[
\vec{u} \cdot \vec{v} =
\]

2. Write out \( (\vec{u} \cdot \vec{v})^2 \) to get 6 terms.
\[
(\vec{u} \cdot \vec{v})^2 =
\]

3. Write the algebraic definition for \( \vec{u} \times \vec{v} \).
\[
\vec{u} \times \vec{v} =
\]

4. Write out \( |\vec{u} \times \vec{v}|^2 \) to get 9 terms.
\[
|\vec{u} \times \vec{v}|^2 =
\]

5. Add \( (\vec{u} \cdot \vec{v})^2 + |\vec{u} \times \vec{v}|^2 \) and cancel some terms.
\[
(\vec{u} \cdot \vec{v})^2 + |\vec{u} \times \vec{v}|^2 =
\]

6. Multiply out \( |\vec{u}|^2 |\vec{v}|^2 \).
\[
|\vec{u}|^2 |\vec{v}|^2 =
\]

7. Are the answers to (5) and (6) equal?