1. Compute the integral \( \int \int x \, dA \) over the region in the first quadrant bounded by \( y = 1 + x^2 \), \( y = 2 + x^2 \), \( y = 3 - x^2 \), and \( y = 5 - x^2 \).

a. Define the curvilinear coordinates \( u \) and \( v \) by \( y = u + x^2 \) and \( y = v - x^2 \). What are the 4 boundaries in terms of \( u \) and \( v \)?

b. Solve for \( x \) and \( y \) in terms of \( u \) and \( v \). Express the results as a position vector.

\[ \vec{R}(u,v) = (x(u,v), y(u,v)) = \]

c. Find the coordinate tangent vectors:

\[ \vec{e}_u = \frac{\partial \vec{R}}{\partial u} = \]

\[ \vec{e}_v = \frac{\partial \vec{R}}{\partial v} = \]

d. Compute the Jacobian determinant:

\[ \frac{\partial (x,y)}{\partial (u,v)} = \]

e. Compute the Jacobian factor:

\[ J = \left| \frac{\partial (x,y)}{\partial (u,v)} \right| = \]

f. Compute the integral:

\[ \int \int x \, dA = \]
Find the Jacobian for spherical coordinates. The position vector is given by
\[ \vec{R}(\rho, \theta, \varphi) = (\rho \sin \varphi \cos \theta, \rho \sin \varphi \sin \theta, \rho \cos \varphi) \]

a. Find the coordinate tangent vectors:
\[ \vec{e}_\rho = \frac{\partial \vec{R}}{\partial \rho} = \]
\[ \vec{e}_\theta = \frac{\partial \vec{R}}{\partial \theta} = \]
\[ \vec{e}_\varphi = \frac{\partial \vec{R}}{\partial \varphi} = \]

b. Compute the Jacobian determinant:
\[ \frac{\partial (x, y, z)}{\partial (\rho, \theta, \varphi)} = \]

c. Compute the Jacobian factor:
\[ J = \left| \frac{\partial (x, y, z)}{\partial (\rho, \theta, \varphi)} \right| = \]