1-8	/40
9	/20
10	/20
11	/20
Total	/100

Name____________________________

MATH 251 Exam 2 Fall 2014

Sections 508 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Compute $\int_0^{\pi}\int_0^{\theta} r^2\,dr\,d\theta$.

a. $\frac{1}{3}\pi\theta^3$

b. $\frac{1}{6}\pi^3$

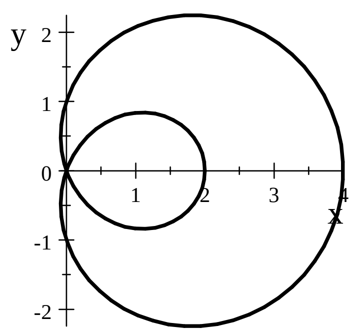
c. $\frac{1}{12}\pi^4$

d. $\frac{1}{20}\pi^5$

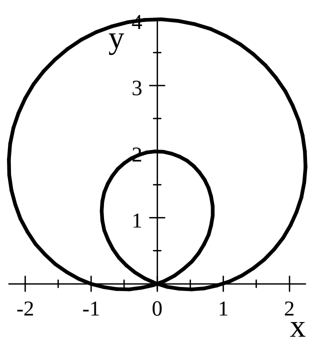
e. $\frac{4}{3}\pi^4$

2. Which of the following is the polar plot of $r = 2 + 3\cos(\theta)$?

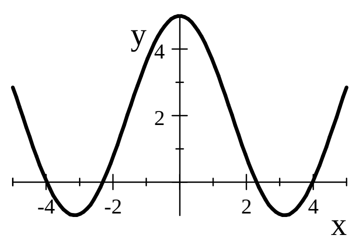
a.



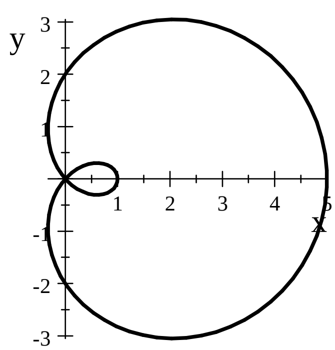
b.



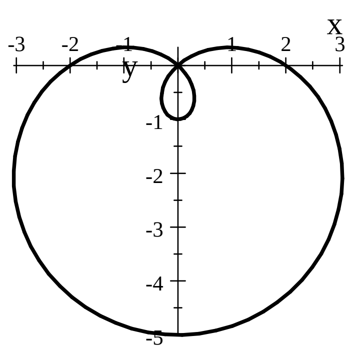
c.



d.



e.



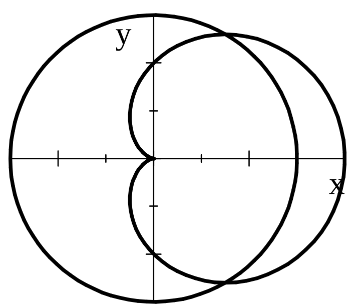
3. A plate has the shape between the parabola $y = x^2$ and the line $y = 4$.
Find its mass if its surface density is $\rho = y$.

a. $\frac{128}{5}$
b. $\frac{64}{5}$
c. $\frac{32}{5}$
d. $\frac{16}{5}$
e. $\frac{8}{5}$

4. A plate has the shape between the parabola $y = x^2$ and the line $y = 4$.
Find the y-component of its center of mass if its surface density is $\rho = y$.

a. $\frac{1024}{7}$
b. $\frac{512}{7}$
c. $\frac{7}{512}$
d. $\frac{20}{7}$
e. $\frac{10}{7}$

5. Find the mass of the region outside the circle $r = 3$ and inside the cardioid $r = 2 + 2\cos\theta$ if the linear mass density is $\rho = \frac{1}{r}$.



a. $2 + \frac{2}{3}\pi$
b. $2 - \frac{2}{3}\pi$
c. $2\sqrt{3} - \frac{2\pi}{3}$
d. $3\sqrt{3} + 2\pi$
e. $3\sqrt{3} - 2\pi$

6. If $\vec{F} = (xyz, 2xyz, 3xyz)$, then $\vec{\nabla} \times \vec{F} =$

a. $(3xz - 2xy, xy - 3yz, 2yz - xz)$

b. $(3xz - 2xy, 3yz - xy, 2yz - xz)$

c. $yz + 2xz + 3xy$

d. $yz - 2xz + 3xy$

e. 0

7. If $\vec{F} = (xyz, 2xyz, 3xyz)$, then $\vec{\nabla} \cdot \vec{F} =$

a. $(3xz - 2xy, xy - 3yz, 2yz - xz)$

b. $(3xz - 2xy, 3yz - xy, 2yz - xz)$

c. $yz + 2xz + 3xy$

d. $yz - 2xz + 3xy$

e. 0

8. Find the average value of $f = z$ on the solid hemisphere $0 \le z \le \sqrt{9 - x^2 - y^2}$.
Note: The average value of a function on a solid is $f_{\text{ave}} = \frac{1}{V} \iiint_V f\,dV$.

a. $\frac{3\pi}{8}$

b. $\frac{\pi}{2}$

c. $\frac{3}{2}$

d. $\frac{1}{2}$

e. $\frac{9}{8}$

Work Out: (Points indicated. Part credit possible. Show all work.)

9. (20 points) A rectangular solid box is sitting on the xy-plane with its upper 4 vertices on the paraboloid $z = 16 - x^2 - 4y^2$. Find the dimensions and volume of the largest such box.

Full credit for solving by Lagrange multipliers.

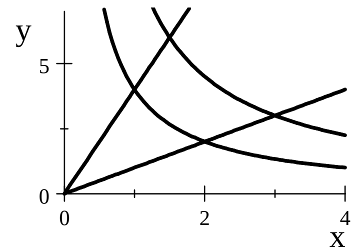
Half credit for solving by Eliminating a Variable.

50% extra credit for solving both ways.

10. (20 points) Compute the integral $\iint y\, dA$ over the region in the first quadrant bounded by

$y = x$, $y = 4x$, $y = \frac{4}{x}$, and $y = \frac{9}{x}$.

Use the following steps:



a. Define the curvilinear coordinates u and v by $y = u^2 x$ and $y = \frac{v^2}{x}$.
Express the coordinate system as a position vector.

$\vec{r}(u,v) =$

b. Find the coordinate tangent vectors:

$\vec{e}_u =$

$\vec{e}_v =$

c. Compute the Jacobian factor:

$J = \left| \frac{\partial(x,y)}{\partial(u,v)} \right| =$

d. Compute the integral:

$\iint y\, dA =$

11. (20 points) Compute the flux $\iint_C \vec{F} \cdot d\vec{S}$ of the vector field $\vec{F} = \left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}, \frac{z}{x^2+y^2} \right)$ outward through the piece of the cylinder $x^2 + y^2 = 9$ between the planes $z = -3$ and $z = x$. Use the following steps:

a. Parametrize the cylinder:

$\vec{R}(z, \theta) =$

b. Find the tangent vectors:

$\vec{e}_z =$

$\vec{e}_\theta =$

c. Find the normal vector:

$\vec{N} =$

d. Fix the orientation of the normal (if necessary):

$\vec{N} =$

e. Evaluate the vector field on the cylinder:

$\vec{F}\left(\vec{R}(z, \theta)\right) =$

f. Evaluate the boundaries on the cylinder:

$z = -3$:

$z = x$:

g. Calculate the flux:

$\iint_C \vec{F} \cdot d\vec{S} =$