Name______________________

MATH 251 Final Exam Fall 2014

Sections 508 P. Yasskin

1-12	/48
13	/20
14	/10
15	/25
Total	/103

Multiple Choice: (4 points each. No part credit.)

1. Find the line of intersection of the planes $3x+2y-4z=8$ and $3x-2y+2z=4$.
This line intersects the xy-plane at

a. $\left(1,-\frac{7}{3},-3\right)$
b. $(4,10,6)$
c. $(1,2,0)$
d. $(2,1,0)$
e. $\left(\frac{22}{9},2,\frac{4}{3}\right)$

2. The radius of a cylinder is currently $r=50$ cm and is increasing at $\dfrac{dr}{dt}=2\,\dfrac{\text{cm}}{\text{min}}$.

Its height is currently $h=100$ cm and decreasing at $\dfrac{dh}{dt}=-4\,\dfrac{\text{cm}}{\text{min}}$.

At what rate is the volume changing?

a. $\dfrac{dV}{dt}=-10\,000\pi$
b. $\dfrac{dV}{dt}=10\,000\pi$
c. $\dfrac{dV}{dt}=20\,000\pi$
d. $\dfrac{dV}{dt}=30\,000\pi$
e. $\dfrac{dV}{dt}=40\,000\pi$

3. Find the tangent plane to the graph of $z = \frac{1}{xy}$ at $(1,2)$. The z-intercept is

a. 0
b. $\frac{1}{2}$
c. $\frac{3}{2}$
d. 2
e. $\frac{5}{2}$

4. Find the tangent plane to the graph of the ellipsoid $x^2 + xy + 4y^2 + z^2 = 20$ at $(3,1,2)$. The z-intercept is

a. 10
b. 20
c. 40
d. 80
e. 160

5. How many critical points does the function $f(x,y) = x^3 - 3xy + y^3$ have?

a. 0
b. 1
c. 2
d. 3
e. infinitely many

6. Duke Skywater is flying the Centenial Eagle through a dangerous polaron field whose density is given by $\rho = x^4 + y^3 + z^2$. If Duke is currently at the point $P = (1,2,3)$ and has velocity $\vec{v} = (4,3,2)$, what is the rate of change of the polaron density as seen by Duke at the current instant?

a. 16
b. 24
c. 32
d. 48
e. 64

7. Duke Skywater is flying the Centenial Eagle through a dangerous polaron field whose density is given by $\rho = x^4 + y^3 + z^2$. If Duke is currently at the point $P = (1,2,3)$ in what **unit vector** direction should he fly to **REDUCE** the polaron density as fast as possible?

a. $\left(\frac{2}{7}, \frac{6}{7}, \frac{3}{7}\right)$
b. $\left(-\frac{2}{7}, -\frac{6}{7}, -\frac{3}{7}\right)$
c. $\left(\frac{2}{7}, -\frac{6}{7}, \frac{3}{7}\right)$
d. $\left(\frac{2}{98}, \frac{6}{98}, \frac{3}{98}\right)$
e. $\left(-\frac{2}{98}, -\frac{6}{98}, -\frac{3}{98}\right)$

8. Find the maximum value of the function $f = x + 2y + 2z$ on the sphere of radius 5 centered at the origin.

a. 5
b. $\frac{25}{3}$
c. 9
d. $\frac{27}{5}$
e. 15

9. A wire has the shape of the parabola $y = x^2$ which may be parametrized as $\vec{r}(t) = (t, t^2)$ from $(0,0)$ to $\left(\sqrt{6}, 6\right)$. Find it mass if its linear density is $\rho = x$.

a. $\frac{1}{4}\ln\left(2\sqrt{6}+5\right)+\frac{5}{2}\sqrt{6}$
b. $\frac{1}{4}\ln\left(2\sqrt{6}+5\right)-\frac{5}{2}\sqrt{6}$
c. $\frac{125}{12}$
d. $\frac{31}{3}$
e. $\frac{21}{2}$

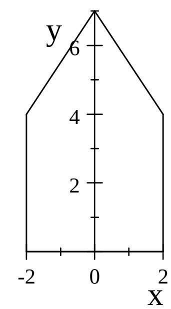
10. Compute $\int \vec{F} \cdot d\vec{s}$ for $\vec{F} = (2xy^2, 2x^2y)$ along the cubic $y = x^3$ from $(1,1)$ to $(2,8)$.
Hint: Use a theorem.

a. 255
b. 256
c. 510
d. 60
e. 66

11. Compute $\oint (3y - 2x^2)\,dx + (3y^2 - 2x)\,dy$

counterclockwise over the complete boundary of the shape at the right, which is a square of side 4 under an isosceles triangle with height 3.

Hint: Use a theorem



a. -110

b. -22

c. 0

d. 22

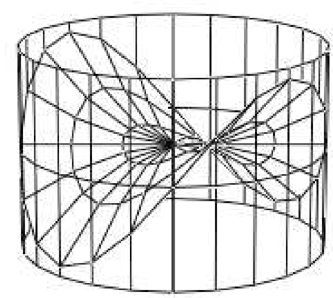
e. 110

12. Compute $\iint_P \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ over the piece of the hyperbolic paraboloid $z = xy$ oriented upward inside the cylinder $x^2 + y^2 = 9$ for the vector field $\vec{F} = \left(-3y, 3x, \sqrt{x^2 + y^2}\right)$.

Note, the paraboloid may be parametrized by

$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2 \sin\theta\cos\theta)$.

Hint: Use a theorem.



a. $\dfrac{27}{2}\pi$

b. 27π

c. 54π

d. 108π

e. 216π

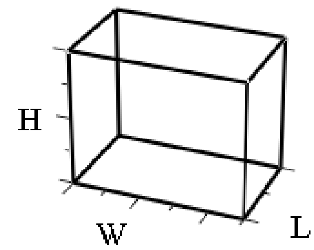
Work Out: (Points indicated. Part credit possible. Show all work.)

13. (20 points) For each integral, plot the region of integration and then compute the integral.

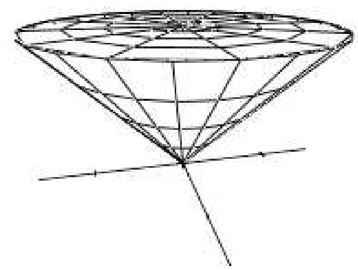
a. $I = \int_0^2 \int_{y^2}^4 \cos(x^{3/2})\, dx\, dy$

b. $J = \int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \arctan\left(\frac{y}{x}\right) dx\, dy$

14. (10 points) An aquarium in the shape of a rectangular solid has a slate bottom costing $\$6$ per ft^2, a glass front costing $\$2$ per ft^2, and aluminum sides and back costing $\$1$ per ft^2. There is no top. Let L be the length front to back, W be the width side to side and H be the height. Write a formula for the total cost, C, and find the dimensions and cost of the cheapest such aquarium if the volume is $V = 36$ ft^3. Do not use decimals.



15. (25 points) Verify Gauss' Theorem $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV = \iint\limits_{\partial V} \vec{F} \cdot d\vec{S}$



for the vector field $\vec{F} = (yz^2, xz^2, (x^2+y^2)z)$ and the solid cone $\sqrt{x^2+y^2} \le z \le 4$.

Be careful with orientations. Use the following steps:

First the Left Hand Side:

a. Compute the divergence:

$\vec{\nabla} \cdot \vec{F} =$

b. Name your coordinate system:

c. Express the divergence and the volume element in those coordinates:

$\vec{\nabla} \cdot \vec{F} =$ $\qquad$ $dV =$

d. Compute the left hand side:

$\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV =$

Second the Right Hand Side:

The boundary surface consists of a cone C and a disk D with appropriate orientations.

e. Parametrize the disk D:

$\vec{R}(r,\theta) =$

f. Compute the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

g. Compute the normal vector:

$\vec{N} =$

h. Evaluate $\vec{F} = (yz^2, xz^2, (x^2+y^2)z)$ on the disk:

$\vec{F}\Big|_{\vec{R}(r,\theta)} =$

i. Compute the dot product:

$\vec{F} \cdot \vec{N} =$

j. Compute the flux through D:

$\iint_D \vec{F} \cdot d\vec{S} =$

The cone C may be parametrized by:
$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$

k. Compute the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

l. Compute the normal vector:

$\vec{N} =$

m. Evaluate $\vec{F} = (yz^2, xz^2, (x^2+y^2)z)$ on the cone:

$\vec{F}\Big|_{\vec{R}(r,\theta)} =$

n. Compute the dot product:

$\vec{F} \cdot \vec{N} =$

o. Compute the flux through C:

$\iint_C \vec{F} \cdot d\vec{S} =$

p. Compute the **TOTAL** right hand side: