1. If \( \mathbf{a} = (4, -2, 1) \) and \( \mathbf{b} = (2, -1, 1) \), then \( |\mathbf{a} - 3\mathbf{b}| = \)

   a. 1
   b. 3 Correct
   c. 5
   d. 9
   e. 13

**Solution:** \( |\mathbf{a} - 3\mathbf{b}| = |(-2, 1, -2)| = \sqrt{4 + 1 + 4} = 3 \)

2. The plot at the right is the contour plot of which function?
   HINT: Where is the level set with value 0?
   a. \( \sin(x)\cos(y) \)
   b. \( \sin(x)\sin(y) \)
   c. \( \cos(x)\cos(y) \)
   d. \( \cos(x)\sin(y) \) Correct
   e. \( \sin(xy) \)

**Solution:** There is a level set at \( x = \frac{\pi}{2} \) but not \( x = 0 \). So it must have a \( \cos(x) \) but not \( \sin(x) \) or \( \sin(xy) \) factor. There is a level set at \( y = 0 \) but not \( y = \frac{\pi}{2} \). So it must have a \( \sin(y) \) but not \( \cos(y) \) factor.
3. Suppose \( \text{proj}\_u \vec{v} = (3, 1) \). Which of the following is inconsistent with this fact?

a. \( \text{proj}\_u \vec{v} = (2, -6) \)

b. \( \text{proj}\_u \vec{v} = (-2, 6) \)

c. \( \vec{u} = (4, -2) \)

d. \( \vec{v} = (6, 2) \)

e. \( \vec{v} = (1, -3) \) Correct

Solution: \( \text{proj}\_u \vec{v} \) must be perpendicular to \( \text{proj}\_u \vec{v} \) and \((2, -6) \cdot (3, 1) = 0 \) and \((-2, 6) \cdot (3, 1) = 0 \). \( \text{proj}\_u \vec{v} \) must be parallel to \( \vec{v} \), and \((6, 2) \) is a multiple but \((1, -3) \) is not.

4. Which of the following is an ellipse in the 1st quadrant tangent to both the x and y-axes?

a. \( 9(x - 3)^2 + 4(y - 2)^2 = 36 \)

b. \( 4(x - 3)^2 + 9(y - 2)^2 = 36 \) Correct

c. \( 4(x - 2)^2 + 9(y - 3)^2 = 36 \)

d. \( 4(x - 3)^2 + 9(y - 2)^2 = 1 \)

e. \( 9(x - 2)^2 + 4(y - 3)^2 = 1 \)

Solution: The centers are \((3, 2)\) in (a), (b) and (d) and \((2, 3)\) in (c) and (e). In (d) and (e) the x and y radii are \(\frac{1}{2}\) and \(\frac{1}{3}\), not enough to reach the axes from the center. In (a) the x radius is 2 which cannot reach the y axis from the center \((3, 2)\). In (c) the y radius is 2 which cannot reach the x axis from the center \((2, 3)\). In (b) the x radius is 3 and the y radius is 2 just matching the distances from the center \((3, 2)\) to the axes.

5. In 3-dimensional space, the equation \( x^2 - 4x - y^2 + 6y + z^2 = 5 \) is

a. a hyperboloid with center \((2, 3, 0)\) and axis \( \vec{r}(t) = (2, 3, t) \).

b. a hyperboloid with center \((2, 3, 0)\) and axis \( \vec{r}(t) = (2, 3 + t, 0) \).

c. a hyperbolic cylinder with axis \( \vec{r}(t) = (2, 3, t) \).

d. a cone with vertex \((2, 3, 0)\) and axis \( \vec{r}(t) = (2, 3 + t, 0) \) Correct

e. two planes which intersect at the line \( \vec{r}(t) = (2, 3 + t, 0) \).

Solution: Complete the squares to get \( (x - 2)^2 - (y - 3)^2 + z^2 = 0 \). This is a cone with axis parallel to the y-axis.
6. If \( \vec{u} \) points SOUTHEAST and \( \vec{v} \) points UP, where does \( \vec{u} \times \vec{v} \) point?

a. DOWN  
b. SOUTHWEST  Correct  
c. WEST  
d. NORTHEAST  
e. NORTHWEST

**Solution:** Point the fingers of your right hand pointing SOUTHEAST with your palm facing UP. Your thumb points SOUTHWEST.

7. Find the intersection of the line \((x,y,z) = (2t, -1 + 2t, 2 + 2t)\) and the plane \(3x - 2y + z = 8\). At this point \(x + y + z = \)

a. -3  
b. -1  
c. 0  
d. 5  
e. 7  Correct

**Solution:** Plug the line into the plane and solve for \(t\):
\[
3(2t) - 2(-1 + 2t) + (2 + 2t) = 8 \quad 4t + 4 = 8 \quad 4t = 4 \quad t = 1
\]
Plug back into the line: \((x,y,z) = (2, -1 + 2, 2 + 2) = (2, 1, 4)\) \(x + y + z = 2 + 1 + 4 = 7\)

8. Compute \(\lim_{h \to 0} \frac{(2x + 2h + 3y)^2 - (2x + 3y)^2}{h}\)

a. \(2x + 3y\)  
b. \(4x + 6y\)  
c. \(6x + 9y\)  
d. \(8x + 12y\)  Correct  
e. \(12x + 18y\)

**Solution:** \(\frac{\partial}{\partial x} (2x + 3y)^2 = 2(2x + 3y)2 = 8x + 12y\)
9. Find the plane tangent to the graph of \( z = x^2 e^{2y} \) at \( (3,0) \). The \( z \)-intercept is

- a. \(-27\)
- b. \(-18\)
- c. \(-9\) Correct
- d. \(9\)
- e. \(18\)

**Solution:**

- \( f(x,y) = x^2 e^{2y} \)
- \( f(3,0) = 9 \)
- \( z = f(3,0) + f_x(3,0)(x-3) + f_y(3,0)(y-0) \)
- \( f_x(x,y) = 2xe^{2y} \)
- \( f_x(3,0) = 6 \)
- \( = 9 + 6(x-3) + 18(y) \)
- \( f_y(x,y) = 2x^2 e^{2y} \)
- \( f_y(3,0) = 18 \)
- \( = 6x + 18y - 9 \)
- \( c = -9 \)

10. If \( S(3,2) = 5 \) and \( \frac{\partial S}{\partial x}(3,2) = -0.3 \) and \( \frac{\partial S}{\partial y}(3,2) = 0.4 \), estimate \( S(3.2,1.7) \).

- a. \(4.82\) Correct
- b. \(4.9\)
- c. \(5.0\)
- d. \(5.1\)
- e. \(5.18\)

**Solution:** The linear approximation says:

- \( S(x,y) \approx S_{tan}(x,y) = S(a,b) + S_x(a,b)(x-a) + S_y(a,b)(y-b) \)

Here \( (x,y) = (3.2,1.7) \) and \( (a,b) = (3,2) \). So

- \( S(3.2,1.7) \approx S(3,2) + S_x(3,2)(.2) + S_y(3,2)(-.3) = 5 - 0.3(.2) + 0.4(-.3) = 4.82 \)
11. A semicircle sits on top of a rectangle of width $2r$ and height $h$. If the radius decreases from 3 cm to 2.97 cm while the height increases from 4 cm to 4.02 cm, use the linear approximation to determine whether the area increases or decreases and by how much.

a. increases by $0.09\pi - 0.12$

b. increases by $0.09\pi + 0.12$

c. increases by $0.09\pi + 0.36$

d. decreases by $0.09\pi + 0.36$

e. decreases by $0.09\pi + 0.12$ Correct

Solution:

\[ A = 2rh + \frac{1}{2}\pi r^2 \]

\[ \Delta A \approx dA = \frac{\partial A}{\partial r}dr + \frac{\partial A}{\partial h}dh = (2h + \pi r)dr + (2r)dh \]

\[ = (2 \cdot 4 + \pi \cdot 3)(-0.03) + (2 \cdot 3)(0.02) = -0.09\pi - 0.12 < 0 \quad \text{decreases} \]

12. The temperature in a room is $T = z^2(2x + 3y)$. Currently, a fly is at $\vec{r} = (4, 3, 2)$ and has velocity $\vec{v} = (3, 2, 1)$. What is the rate of change of the temperature as seen by the fly?

a. 16

b. 116 Correct

c. 64

d. 164

e. 264

Solution:

\[ \frac{dT}{dt} = \frac{\partial T}{\partial x} \frac{dx}{dt} + \frac{\partial T}{\partial y} \frac{dy}{dt} + \frac{\partial T}{\partial z} \frac{dz}{dt} = (2z^2)v_1 + (3z^2)v_2 + (2z(2x + 3y))v_3 \]

\[ = (2 \cdot 2^2)3 + (3 \cdot 2^2)2 + (2 \cdot 2(2 \cdot 4 + 3 \cdot 3))1 = 116 \]
13. (16 points) For the parametric curve \( \vec{r}(t) = \left( \frac{2}{t}, 6t, 3t^3 \right) \) compute each of the following:

- a. velocity \( \vec{v} \)
  
  **Solution:** \( \vec{v} = \left( \frac{-2}{t^2}, 6, 9t^2 \right) \)

- b. speed \( |\vec{v}| \)
  
  **HINT:** The quantity inside the square root is a perfect square.
  
  **Solution:** \( |\vec{v}| = \sqrt{\frac{4}{t^4} + 36 + 81t^4} = \frac{2}{t^2} + 9t^2 \)

- c. arc length \( L = \int_{(2,6,3)}^{(1,12,24)} ds \)
  
  **Solution:** \( L = \int_{1}^{2} |\vec{v}|dt = \int_{1}^{2} \left( \frac{2}{t^2} + 9t^2 \right)dt = \left[ -\frac{2}{t} + 3t^3 \right]_1 = -(1 + 24) - (-2 + 3) = 22 \)

- d. acceleration \( \vec{a} \)
  
  **Solution:** \( \vec{a} = \left( \frac{4}{t^3}, 0, 18t \right) \)

- e. unit binormal \( \hat{B} \)
  
  **Solution:** \( \vec{v} \times \vec{a} = \begin{vmatrix} i & j & k \\ \frac{-2}{t^2} & 6 & 9t^2 \\ \frac{4}{t^3} & 0 & 18t \end{vmatrix} = i(108t) - j\left( \frac{-36}{t} - \frac{36}{t} \right) + k\left( \frac{-24}{t^3} \right) = \left( 108t, \frac{72}{t}, \frac{-24}{t^3} \right) \)

  \[
  |\vec{v} \times \vec{a}| = 12\sqrt{81t^2 + 36t^2 + \frac{4}{t^6}} = 12\left( 9t + \frac{2}{t^3} \right) = \frac{12(9t^4 + 2)}{t^3} = \frac{9t^4}{9t^4 + 2}, \frac{9t^4}{9t^4 + 2}, \frac{-9t^4 + 2}{9t^4 + 2} \]

  \[
  \hat{B} = \frac{\vec{v} \times \vec{a}}{|\vec{v} \times \vec{a}|} = \frac{\frac{4}{t^3}}{9t^4 + 2}, \frac{6t^2}{9t^4 + 2}, \frac{-9t^4 + 2}{9t^4 + 2} \]

- f. tangential acceleration \( a_T \)
  
  **Solution:** \( a_T = \frac{d|\vec{v}|}{dt} = \frac{d}{dt} \left( \frac{2}{t^2} + 9t^2 \right) = \frac{-4}{t^3} + 18t \)

14. (12 points) A wire has the shape of the parametric curve \( \vec{r}(t) = \left( \frac{2}{t}, 6t, 3t^3 \right) \) between \( 2, 6 \) and \( 1, 12, 24 \). Find the mass of the wire if the linear mass density is \( \rho = \frac{1}{12}xyz \).

Don’t simplify the answer.

**Solution:** \( \vec{v} = \left( \frac{-2}{t^2}, 6, 9t^2 \right) \) \( |\vec{v}| = \frac{2}{t^2} + 9t^2 \) \( \rho = \frac{1}{12}xyz = \frac{1}{12} \left( \frac{2}{t} \right)(6t)(3t^3) = 3t^3 \)

\[
M = \int_{(2,6,3)}^{(1,12,24)} \rho ds = \int_{1}^{2} \frac{1}{12}xyz|\vec{v}|dt = \int_{1}^{2} 3t^3 \left( \frac{2}{t^2} + 9t^2 \right)dt = \int_{1}^{2} (6t + 27t^5) dt = \left[ 3t^2 + \frac{9t^6}{2} \right]_1 = \left( 12 + \frac{9 \cdot 2^6}{2} \right) - \left( 3 + \frac{9}{2} \right) = \frac{585}{2} \]

15. (12 points) A mass slides along a wire which has the shape of the parametric curve \( \vec{r}(t) = \left( \frac{2}{t}, 6t, 3t^3 \right) \) between \( 2, 6, 3 \) and \( 1, 12, 24 \) under the action of the force \( \vec{F} = (z,y,x) \).

Find the work done by the force.

**Solution:** \( \vec{F} = (z,y,x) = (3t^3, 6t, \frac{2}{t}) \) \( \vec{v} = \left( \frac{-2}{t^2}, 6, 9t^2 \right) \)

\[
\vec{F} \cdot \vec{v} = 3t^3 \frac{-2}{t^2} + 6t^2 + \frac{2}{t}9t^2 = -6t + 36t + 18t = 48t 
\]

\[
W = \int_{(2,6,3)}^{(1,12,24)} \vec{F} \cdot ds = \int_{1}^{2} \vec{F} \cdot \vec{v}dt = \int_{1}^{2} 48t dt = \left[ 24t^2 \right]_1 = 24(4 - 1) = 72 
\]