Name________________________

1-9	/45
10	/30
11	/15
12	/15
Total	/105

MATH 251 Exam 2A Fall 2015

Sections 511/512 (circle one) P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Find the volume below the surface $z = x + 2y$ above the region in the xy-plane between $y = x^2$ and $y = 3x$.

a. $\frac{-71}{20}$

b. $\frac{71}{20}$

c. $\frac{972}{5}$

d. $\frac{783}{20}$

e. $\frac{1944}{5}$

2. The temperature on a hot plate with dimensions $-4\pi \le x \le 4\pi$ and $0 \le y \le 6$ is $T = x^2 y + \cos^2 x$. Find the average temperature.

a. 24π

b. $768\pi^3$

c. $768\pi^3 + 24\pi$

d. $16\pi^2 + \frac{1}{2}$

e. $16\pi^2$

3. Find the centroid of the region above $y = 2x^2$ below $y = 18$.

a. $\left(0, \frac{27}{5}\right)$

b. $\left(0, \frac{36}{5}\right)$

c. $\left(0, \frac{54}{5}\right)$

d. $\left(0, \frac{1944}{5}\right)$

e. $\left(0, \frac{3888}{5}\right)$

4. Find all critical points of the function $f(x,y) = 4x^2 + 9y^2 + \frac{432}{xy}$. Select from:

$A = (2,3)$ $B = (-2,3)$ $C = (2,-3)$ $D = (-2,-3)$

$E = (3,2)$ $F = (-3,2)$ $G = (3,-2)$ $H = (-3,-2)$

Note $432 = 2^4 3^3$

a. A, B, C, D

b. E, F, G, H

c. A, D

d. B, C

e. E, H

5. Select all of the following statements which are consistent with this countour plot?

A. There is a local maximum at $(1,1)$.

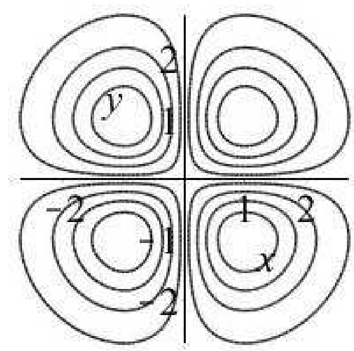
B. There is a local minimum at $(1,1)$.

C. There is a saddle point at $(1,1)$.

D. There is a local maximum at $(0,0)$.

E. There is a local minimum at $(0,0)$.

F. There is a saddle point at $(0,0)$.



a. A,B,D,E

b. C,F

c. C,D,E

d. A,B,F

6. The function $f = \frac{4}{x} - \frac{2}{y} - xy$ has a critical point at $(x,y) = (2,-1)$. Use the Second Derivative Test to classify this critical point.

a. Local Minimum

b. Local Maximum

c. Inflection Point

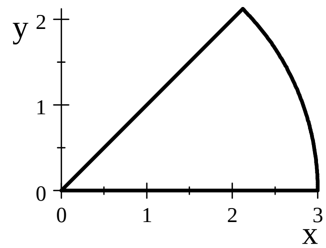
d. Saddle Point

e. Test Fails

7. Find the mass of the $\frac{1}{8}$ of the circle

$x^2+y^2 \le 9$ for $0 \le y \le x$

if the surface density is $\delta = x$.



a. 9

b. $\frac{9}{2}$

c. $9\sqrt{2}$

d. $\frac{9}{\sqrt{2}}$

e. $\frac{9}{2\sqrt{2}}$

8. Find the y-component of the center of mass of the $\frac{1}{8}$ of the circle $x^2+y^2 \le 9$ for $0 \le y \le x$ if the surface density is $\delta = x$.

a. $\frac{9\sqrt{2}}{16}$

b. $\frac{9\sqrt{2}}{8}$

c. $\frac{81}{16}$

d. $\frac{81}{32}$

e. $\frac{1}{\sqrt{2}}$

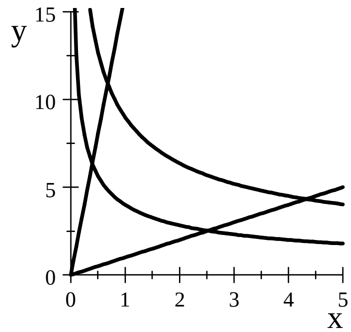
9. Compute $\iint y\,dA$ over the “diamond” shaped region in the first quadrant bounded by

$xy^2 = 8 \qquad xy^2 = 27 \qquad y = x \qquad y = 8x$

HINTS: Use the coordinates

$x = \dfrac{u}{v^2} \qquad y = uv$

Find the boundaries and Jacobian.



a. $19\ln 2$

b. $80\ln 2$

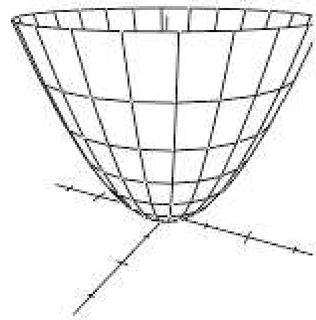
c. $15\ln 3$

d. $65\ln 2$

e. $65\ln 3$

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (30 points) Consider the piece of the paraboloid surface $z = 2x^2 + 2y^2$ for $z \le 18$.



- Find the mass of the paraboloid if the surface mass density is $\delta = \dfrac{3z}{x^2 + y^2}$.
- Find the flux of the electric field $\vec{E} = \left(\dfrac{x}{x^2 + y^2}, \dfrac{y}{x^2 + y^2}, 0 \right)$ down and out of the paraboloid.

Parametrize the surface as $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, 2r^2)$ and follow these steps:

a. Find the coordinate tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

b. Find the normal vector and check its orientation.

$\vec{N} =$

c. Find the length of the normal vector.

$\left|\vec{N}\right| =$

d. Evaluate the density $\delta = \dfrac{3z}{x^2 + y^2}$ on the paraboloid.

$\delta\left(\vec{R}(r,\theta)\right) =$

e. Compute the mass.

$M =$

f. Evaluate the electric field $\vec{E} = \left(\dfrac{x}{x^2 + y^2}, \dfrac{y}{x^2 + y^2}, 0\right)$ on the paraboloid.

$\vec{E}\left(\vec{R}(r,\theta)\right) =$

g. Compute the flux.

$\iint \vec{E} \cdot d\vec{S} =$

11. (15 points) Find the volume of the largest rectangular solid which sits on the xy-plane and has its upper 4 vertices on the paraboloid $z + x^2 + 4y^2 = 64$.

12. (15 points) Draw the region of integration and compute $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3 + 1}\, dx\, dy$.