Name________________________ ID__________________

1-10	/50
11	/30
12	/ 5
13	/20
Total	/105

MATH 251 Final Exam Fall 2015

Sections 511 Version A P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. A triangle has vertices $A = (3,2,4)$, $B = (3,4,6)$ and $C = (6,-1,4)$. Find its area.

a. 3

b. $2\sqrt{3}$

c. $3\sqrt{3}$

d. $6\sqrt{3}$

e. 6

2. Find the tangent plane to the graph of $z = x^3y + y^3x^2$ at $(x,y) = (1,2)$.
Where does it cross the z-axis?

a. -38

b. -14

c. -10

d. 10

e. 38

3. Find the tangent plane to the graph of hyperbolic paraboloid

$$-4(x-3)^2 - 4(y-1)^2 + 9(z-2)^2 = 1$$

at the point $(4,2,1)$. Where does it cross the z-axis?

a. $-\frac{11}{9}$

b. $-\frac{11}{3}$

c. 0

d. $\frac{11}{3}$

e. $\frac{11}{9}$

4. Sketch the region of integration for the integral $\int_0^1 \int_{x^2}^1 x\sin(y^2)\, dy\, dx$ in problem (12), then select its value here:

a. $\frac{1}{4}\cos 1$

b. $\frac{1}{4}(1-\cos 1)$

c. $\frac{1}{4}(\cos 1 - 1)$

d. $1-\cos 1$

e. $\cos 1 - 1$

5. Find the point on the curve $\vec{r}(t) = (50t - 3t^2, 25t - 4t^2)$ at which the speed is a minimum.

a. $(20, -15)$

b. $(80, -65)$

c. $(80, 65)$

d. $(175, 25)$

e. $(325, 225)$

6. The surface of the Death star is a sphere of radius 2 with a hole cut out of one end, which we will take as centered at the south pole. It may be parametrized by

$$R(\phi,\theta) = (2\sin\phi\cos\theta, 2\sin\phi\sin\theta, 2\cos\phi)$$

for $0 \le \phi \le \frac{2\pi}{3}$. Find the surface area.

a. $A = 12\pi$

b. $A = 6\pi$

c. $A = 3\pi$

d. $A = 2\pi$

e. $A = \pi$

7. Consider the solid below the cone given in cylindrical coordinates by $z = 4 - r$ above the xy-plane. Find the z-component of its centroid.

a. π

b. $\frac{1}{2}$

c. 1

d. $\frac{32\pi}{3}$

e. $\frac{64\pi}{3}$

8. Compute $\oint_{\partial R} \vec{\nabla} f \cdot d\vec{s}$ for $f = ye^x$

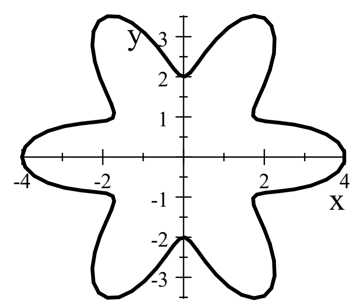
counterclockwise around

the polar curve

$r = 3 + \cos(6\theta)$

shown at the right.

Hint: Use a Theorem.



a. 0

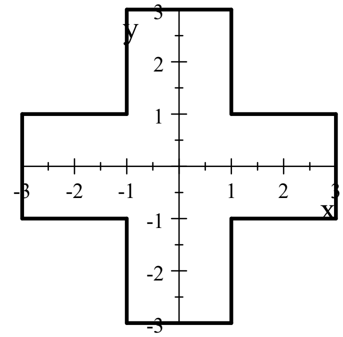
b. $6e^3$

c. $3e^6$

d. $6e^6 - 3e^3$

e. $3e^3 - 6e^6$

9. Compute $\oint_{\partial P} \vec{F} \cdot d\vec{s}$ for $\vec{F} = (5x^3 + 7y, 4x - 6y^4)$ counterclockwise around the complete boundary of the plus sign shown at the right. Hint: Use a Theorem.



a. 20

b. 3

c. -3

d. -30

e. -60

10. Compute $\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ over the cone $z = \sqrt{x^2 + y^2}$ for $z \leq 4$ oriented down and out for $\vec{F} = (y\sqrt{z}, -x\sqrt{z}, \sqrt{z})$.
Hint: Use a Theorem.

a. 4

b. 8π

c. 16

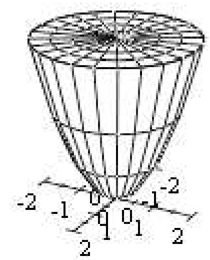
d. 32

e. 64π

Work Out: (Points indicated. Part credit possible. Show all work.)

11. (30 points) Verify Gauss' Theorem $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV = \iint\limits_{\partial V} \vec{F} \cdot d\vec{S}$

for the vector field $\vec{F} = \left(x^3 z, y^3 z, (x^2+y^2)^3\right)$ and the solid between the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.
Be careful with orientations. Use the following steps:



Left Hand Side:

a. Compute the divergence:

$\vec{\nabla} \cdot \vec{F} =$

b. Express the divergence and the volume element in the appropriate coordinate system:

$\vec{\nabla} \cdot \vec{F} =$ $\qquad\qquad$ $dV =$

c. Compute the left hand side:

$\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV =$

Right Hand Side:

The boundary surface consists of a paraboloid P and a disk D with appropriate orientations.

d. Parametrize the disk D:

$\vec{R}(r,\theta) =$

e. Compute the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

f. Compute the normal vector:

$\vec{N} =$

g. Evaluate $\vec{F} = \left(x^3 z, y^3 z, (x^2+y^2)^3\right)$ on the disk:

$\vec{F}\Big|_{\vec{R}(r,\theta)} =$

h. Compute the dot product:

$\vec{F} \bullet \vec{N} =$

i. Compute the flux through D:

$\iint_D \vec{F} \bullet d\vec{S} =$

The paraboloid P may be parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$

j. Compute the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

k. Compute the normal vector:

$\vec{N} =$

l. Evaluate $\vec{F} = \left(x^3 z, y^3 z, (x^2+y^2)^3\right)$ on the paraboloid:

$\vec{F}\big|_{\vec{R}(r,\theta)} =$

m. Compute the dot product:

$\vec{F} \bullet \vec{N} =$

n. Compute the flux through P:

Hints: Use these integrals as needed, without proof:

$$\int_0^{2\pi} \sin^2\theta\, d\theta = \int_0^{2\pi} \cos^2\theta\, d\theta = \pi \qquad \int_0^{2\pi} \sin^4\theta\, d\theta = \int_0^{2\pi} \cos^4\theta\, d\theta = \frac{3}{4}\pi \qquad \int_0^{2\pi} \sin^2\theta \cos^2\theta\, d\theta = \frac{1}{4}\pi$$

$\iint_P \vec{F} \bullet d\vec{S} =$

o. Compute the **TOTAL** right hand side:

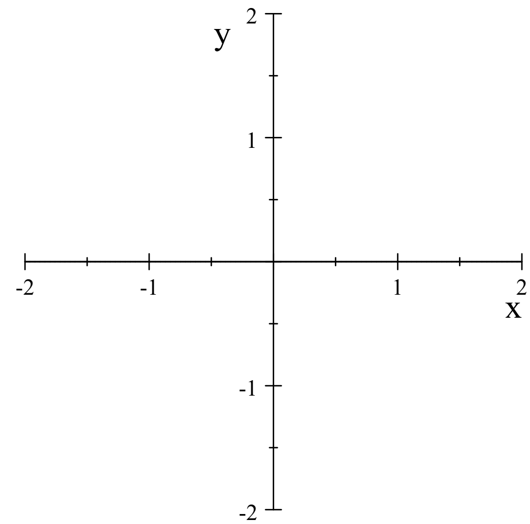
$\iint_{\partial V} \vec{F} \bullet d\vec{S} =$

12. (5 points) Sketch the region of integration for the integral

$$\int_0^1 \int_{x^2}^1 x \sin(y^2)\, dy\, dx.$$

Shade in the region.

You computed its value in problem (4).



13. (20 points) Ham Duet is flying the Millenium Eagle through the galaxy. His current galactic position is $P = (1,4,4)$ lightyears. He is passing through a deadly polaron field whose density is $\delta = x^2 - yz$ polarons/lightyear^3.

a. If his current velocity is $\vec{v} = \langle 0.3, 0.2, 0.1 \rangle$ lightyears/year, at what rate does he see the polaron density changing?

b. Ham decides to change his velocity to get out of the polaron field. If the Millenium Eagle's maximum speed is 0.9 lightyears/year, with what velocity should Ham travel to reduce the polaron's density as fast as possible?