Name________________________ ID__________________

1-10	/50
11	/30
12	/ 5
13	/20
Total	/105

MATH 251 Final Exam Fall 2015

Sections 511 Version B P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. A triangle has vertices $A = (3,2,4)$, $B = (3,4,6)$ and $C = (6,-1,4)$. Find the angle at A.

a. $30°$

b. $45°$

c. $60°$

d. $120°$

e. $150°$

2. Find the tangent plane to the graph of $z = x^2y + y^3x^3$ at $(x,y) = (2,1)$.
Where does it cross the z-axis?

a. -72

b. -48

c. -12

d. 12

e. 48

3. Find the tangent plane to the graph of hyperbolic paraboloid

$$9(x-3)^2 - 4(y-1)^2 - 4(z-2)^2 = 1$$

at the point $(2, 2, 3)$. Where does it cross the z-axis?

a. $\frac{19}{2}$

b. 19

c. 0

d. -19

e. $-\frac{19}{2}$

4. Sketch the region of integration for the integral $\int_0^{2\sqrt{2}} \int_x^{\sqrt{16-x^2}} e^{x^2+y^2}\, dy\, dx$ in problem (12), then select its value here:

a. $\frac{\pi}{4}(e^{16} - 1)$

b. $\frac{\pi}{4}(e^{8} - 1)$

c. $\frac{\pi}{8}(e^{16} - 1)$

d. $\frac{\pi}{8}(e^{8} - 1)$

e. $\frac{\pi}{16}(e^{16} - 1)$

5. Find the line perpendicular to the curve $x^3 + y^3 - xy = 7$ at the point $(2, 1)$.

a. $(11 - 2t, 1 - t)$

b. $(2 + t, 1 + 11t)$

c. $(2 + t, 1 - 11t)$

d. $(2 + 11t, 1 - t)$

e. $(2 + 11t, 1 + t)$

6. Consider the surface of the cone given in cylindrical coordinates by $z = 4 - r$ above the xy-plane. It may be parametrized by

$$R(r,\theta) = (r\cos\theta, r\sin\theta, 4 - r).$$

Find the z-component of its centroid.

a. $\frac{1}{3}$

b. $\frac{2}{3}$

c. $\frac{4}{3}$

d. $\frac{32\sqrt{2}\,\pi}{3}$

e. $16\sqrt{2}\,\pi$

7. Death star is basically a spherical shell with a hole cut out of one end, which we will take as centered at the south pole. In spherical coordinates, it fills the region between $1 \le \rho \le 4$ and $0 \le \phi \le \frac{2\pi}{3}$. Find the volume.

a. $V = 21\pi$

b. $V = 63\pi$

c. $V = 84\pi$

d. $V = 105\pi$

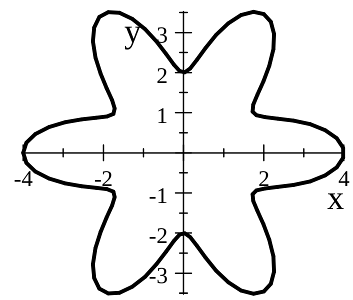
e. $V = 126\pi$

8. Compute $\oint_{\partial R} \vec{\nabla} f \bullet d\vec{s}$ for $f = ye^x$ counterclockwise around the polar curve

$r = 3 + \cos(6\theta)$

shown at the right.

Hint: Use a Theorem.



a. $6e^3$

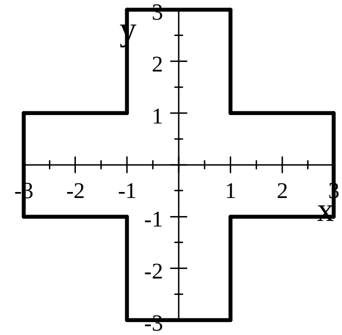
b. $3e^6$

c. $6e^6 - 3e^3$

d. $3e^3 - 6e^6$

e. 0

9. Compute $\oint_{\partial P} \vec{F} \cdot d\vec{s}$ for $\vec{F} = (3x^3 + 5y, 7x - 4y^4)$ counterclockwise around the complete boundary of the plus sign shown at the right.
Hint: Use a Theorem.



a. -40

b. -20

c. 20

d. 40

e. 240

10. Compute $\iint_P \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ over the paraboloid $z = 4(x^2 + y^2)$ for $z \le 16$ oriented down and out for $\vec{F} = (y\sqrt{z}, -x\sqrt{z}, \sqrt{z})$.
Hint: Use a Theorem.

a. 64

b. 32π

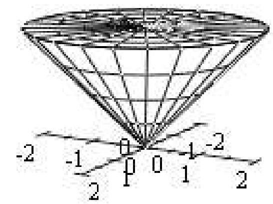
c. 16

d. 8π

e. 4

Work Out: (Points indicated. Part credit possible. Show all work.)

11. (30 points) Verify Gauss' Theorem $\iiint_V \vec{\nabla} \cdot \vec{F}\, dV = \iint_{\partial V} \vec{F} \cdot d\vec{S}$ for the vector field $\vec{F} = \left(x^3 z, y^3 z, (x^2+y^2)^2\right)$ and the solid between the cone $z = \sqrt{x^2+y^2}$ and the plane $z = 2$.



Be careful with orientations. Use the following steps:

Left Hand Side:

a. Compute the divergence:

$\vec{\nabla} \cdot \vec{F} =$

b. Express the divergence and the volume element in the appropriate coordinate system:

$\vec{\nabla} \cdot \vec{F} =$ $\qquad\qquad$ $dV =$

c. Compute the left hand side:

$\iiint_V \vec{\nabla} \cdot \vec{F}\, dV =$

Right Hand Side:

The boundary surface consists of a cone C and a disk D with appropriate orientations.

d. Parametrize the disk D:

$\vec{R}(r,\theta) =$

e. Compute the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

f. Compute the normal vector:

$\vec{N} =$

g. Evaluate $\vec{F} = \left(x^3 z, y^3 z, (x^2+y^2)^2\right)$ on the disk:

$\vec{F}\big|_{\vec{R}(r,\theta)} =$

h. Compute the dot product:

$\vec{F} \cdot \vec{N} =$

i. Compute the flux through D:

$\iint_D \vec{F} \cdot d\vec{S} =$

The cone C may be parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$

j. Compute the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

k. Compute the normal vector:

$\vec{N} =$

l. Evaluate $\vec{F} = \left(x^3 z, y^3 z, (x^2 + y^2)^2\right)$ on the cone:

$\vec{F}\Big|_{\vec{R}(r,\theta)} =$

m. Compute the dot product:

$\vec{F} \cdot \vec{N} =$

n. Compute the flux through C:
Hints: Use these integrals as needed, without proof:

$$\int_0^{2\pi} \sin^2\theta\, d\theta = \int_0^{2\pi} \cos^2\theta\, d\theta = \pi \qquad \int_0^{2\pi} \sin^4\theta\, d\theta = \int_0^{2\pi} \cos^4\theta\, d\theta = \frac{3}{4}\pi \qquad \int_0^{2\pi} \sin^2\theta \cos^2\theta\, d\theta = \frac{1}{4}\pi$$

$\iint_C \vec{F} \cdot d\vec{S} =$

o. Compute the **TOTAL** right hand side:

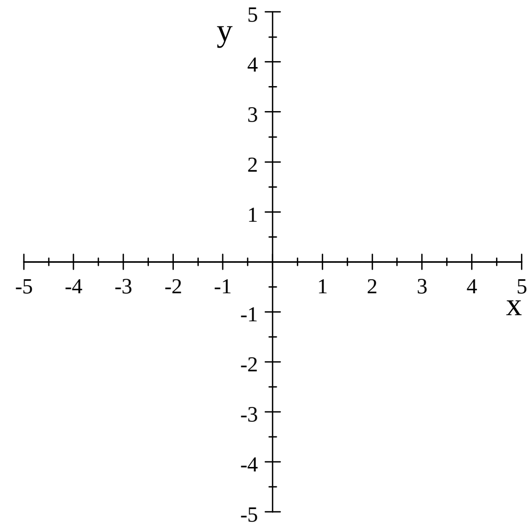
$\iint_{\partial V} \vec{F} \cdot d\vec{S} =$

12. (5 points) Sketch the region of integration for the integral

$$\int_0^{2\sqrt{2}} \int_x^{\sqrt{16-x^2}} e^{x^2+y^2}\, dy\, dx.$$

Shade in the region.

You computed its value in problem (4).



13. (20 points) A cardboard box needs to hold 96 cm^3. The cardboard for the vertical sides costs 12¢ per cm^2 while the thicker bottom costs 36¢ per cm^2. There is no top. What are the length, width, height and cost of the box which costs the least?