1-10	/50
11	/30
12	/ 5
13	/20
Total	/105

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MATH 251 Final Exam Fall 2015

Sections 512 Version B P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. A triangle has vertices $A = (2,2,1)$, $B = (3,4,2)$ and $C = (2,5,4)$. Find its area.

a. $\frac{3}{2}$

b. $\frac{3}{2}\sqrt{3}$

c. $\sqrt{3}$

d. $3\sqrt{3}$

e. 3

2. Find the tangent plane to the graph of $z = x^3y + y^3x^2$ at $(x,y) = (1,2)$.
Where does it cross the z-axis?

a. -38

b. -14

c. -10

d. 10

e. 38

3. Find the tangent plane to the graph of hyperbolic paraboloid

$$-4(x-3)^2 - 4(y-1)^2 + 9(z-2)^2 = 1$$

at the point $(4,2,1)$. Where does it cross the z-axis?

a. $-\frac{11}{9}$

b. $-\frac{11}{3}$

c. 0

d. $\frac{11}{3}$

e. $\frac{11}{9}$

4. Queen Lean is flying the Millenium Eagle through the galaxy. Her current galactic position is $P = (4,3,1)$ lightyears and her current velocity is $\vec{v} = \langle 0.1, 0.2, 0.3 \rangle$ lightyears/year. She is passing through a deadly polaron field whose density δ is related to the dark energy intensity I and the dark matter pressure P by $\delta = IP$.

She measures the dark energy intensity and its gradient are currently

$$I = 7 \text{ lumens and } \vec{\nabla} I = \langle 2,3,1 \rangle \text{ lumens /lightyear}$$

She measures the dark matter pressure and its gradient are currently

$$P = 0.8 \text{ dynes/lightyear}^2 \text{ and } \vec{\nabla} P = \langle 0.4, 0.1, 0.2 \rangle \text{ dynes/lightyear}^3$$

At what rate does she see the polaron density changing?

a. $\frac{d\delta}{dt} = -7.724$

b. $\frac{d\delta}{dt} = -1.22$

c. $\frac{d\delta}{dt} = -1.06$

d. $\frac{d\delta}{dt} = 1.06$

e. $\frac{d\delta}{dt} = 7.724$

5. Sketch the region of integration for the integral $\int_0^2 \int_y^{\sqrt{8-y^2}} e^{x^2+y^2}\,dx\,dy$ in problem (12), then select its value here:

a. $\frac{\pi}{4}(e^8 - 1)$

b. $\frac{\pi}{8}(e^{16} - 1)$

c. $\frac{\pi}{8}(e^8 - 1)$

d. $\frac{\pi}{16}(e^{16} - 1)$

e. $\frac{\pi}{16}(e^8 - 1)$

6. The surface of the Death star is a sphere of radius 2 with a hole cut out of one end, which we will take as centered at the south pole. It may be parametrized by

$$R(\phi,\theta) = (2\sin\phi\cos\theta, 2\sin\phi\sin\theta, 2\cos\phi)$$

for $0 \le \phi \le \frac{2\pi}{3}$. Find the surface area.

a. $A = \pi$

b. $A = 2\pi$

c. $A = 3\pi$

d. $A = 6\pi$

e. $A = 12\pi$

7. Consider the solid below the cone given in cylindrical coordinates by $z = 4 - r$ above the xy-plane. Its temperature is $T = z$. Find its average temperature.

a. π

b. 1

c. $\frac{1}{2}$

d. $\frac{32\pi}{3}$

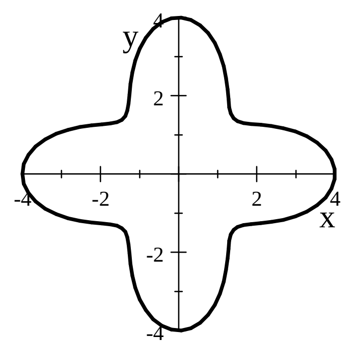
e. $\frac{64\pi}{3}$

8. Compute $\oint_{\partial R} \vec{\nabla} f \cdot d\vec{s}$ for $f = xe^y$ counterclockwise around the polar curve

$r = 3 + \cos(4\theta)$

shown at the right.

Hint: Use a Theorem.



a. 0

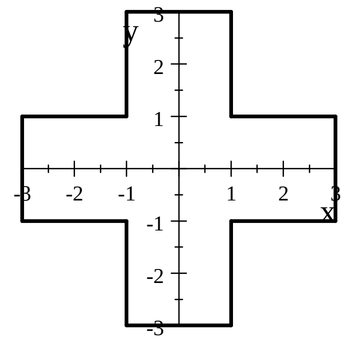
b. $3e^4$

c. $4e^3$

d. $3e^3 - 4e^4$

e. $4e^4 - 3e^3$

9. Compute $\oint_{\partial P} \vec{F} \cdot d\vec{s}$ for $\vec{F} = (8x^3 + 5y, x - 5y^4)$ counterclockwise around the complete boundary of the plus sign shown at the right.
Hint: Use a Theorem.



a. -20

b. -40

c. -80

d. 20

e. 80

10. Compute $\iint_{\partial V} \vec{F} \cdot d\vec{S}$ over the complete surface of the hemisphere $0 \le z \le \sqrt{4 - x^2 - y^2}$ oriented outward,for $\vec{F} = \left(x^3 z, y^3 z, \frac{3}{4} z^4\right)$
Hint: Use a Theorem.

a. -128π

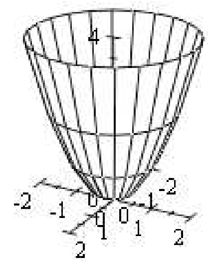
b. -64π

c. -32π

d. 32π

e. 64π

Work Out: (Points indicated. Part credit possible. Show all work.)



11. (30 points) Verify Stokes' Theorem $\iint\limits_{P} \vec{\nabla} \times \vec{F} \bullet d\vec{S} = \oint\limits_{\partial P} \vec{F} \bullet d\vec{s}$

for the vector field $\vec{F} = (yz^2, -xz^2, z^3)$ and the surface of the paraboloid $z = x^2 + y^2$ for $z \le 4$, oriented down and out. Be careful with orientations. Use the following steps:

Left Hand Side:

The paraboloid, P, may be parametrized as $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$

a. Compute the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

b. Compute the normal vector:

$\vec{N} =$

c. Compute the curl of the vector field $\vec{F} = (yz^2, -xz^2, z^3)$:

$\vec{\nabla} \times \vec{F} =$

d. Evaluate the curl of $\vec{F}$ on the paraboloid:

$\vec{\nabla} \times \vec{F}\Big|_{\vec{R}(r,\theta)} =$

e. Compute the dot product of the curl of $\vec{F}$ and the normal $\vec{N}$.

$\vec{\nabla} \times \vec{F} \bullet \vec{N} =$

f. Compute the left hand side:

$\iint\limits_{P} \vec{\nabla} \times \vec{F} \bullet d\vec{S} =$

Right Hand Side:

g. Parametrize the circle, ∂C:

$\vec{r}(\theta) =$

h. Find the tangent vector on the curve:

$\vec{v} =$

i. Evaluate $\vec{F} = (yz^2, -xz^2, z^3)$ on the circle:

$\vec{F}\Big|_{\vec{r}(\theta)} =$

j. Compute the dot product of $\vec{F}$ and the tangent vector $\vec{v}$:

$\vec{F} \cdot \vec{v} =$

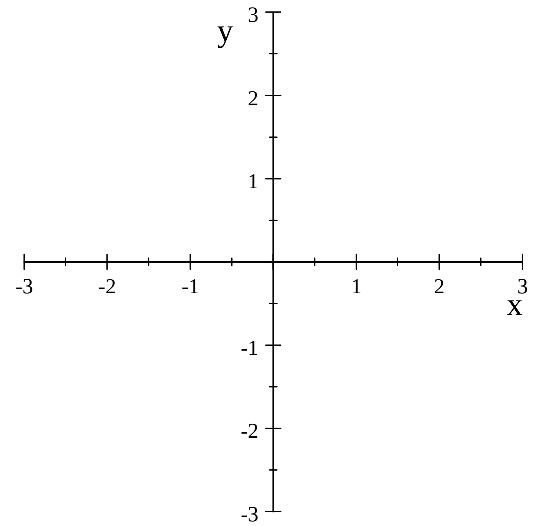
k. Compute the right hand side:

$$\int_{\partial P} \vec{F} \cdot d\vec{s} =$$

12. (5 points) Sketch the region of integration for the integral

$$\int_0^2 \int_y^{\sqrt{8-y^2}} e^{x^2+y^2}\, dx\, dy.$$

You computed its value in problem (5).



13. (20 points) Duke Skywater is flying the Millenium Eagle through the galaxy. His current galactic position is $P = (4,4,1)$ lightyears. He is passing through a deadly polaron field whose density is $\delta = xy - z^2$ polarons/lightyear^3.

a. If his current velocity is $\vec{v} = \langle 0.3, 0.2, 0.1 \rangle$ lightyears/year, at what rate does he see the polaron density changing?

b. Duke decides to change his velocity to get out of the polaron field. If the Millenium Eagle's maximum speed is 0.9 lightyears/year, with what velocity should Duke travel to reduce the polaron's density as fast as possible?