1-9	/63
10	/20
11	/24
Total	/107

Name___________________________

MATH 251 Exam 2A Fall 2016

Sections 504 P. Yasskin

Multiple Choice: (7 points each. No part credit.)

1. The function $f = xy - \frac{2}{x} + \frac{4}{y}$ has a critical point at $(x,y) = (1,-2)$.
Use the Second Derivative Test to classify this critical point.

- **a**. Local Minimum
- **b**. Local Maximum
- **c**. Inflection Point
- **d**. Saddle Point
- **e**. Test Fails

2. Find the volume of the solid under $z = 2x^2y$ above the region in the xy-plane between $y = x$ and $y = x^2$.

- **a**. $\frac{2}{35}$
- **b**. $\frac{35}{12}$
- **c**. $\frac{12}{35}$
- **d**. $\frac{1}{35}$
- **e**. $\frac{1}{12}$

3. Compute $\iint \sin(x^2)\,dx\,dy$ over the triangle with vertices $(0,0)$, $(\sqrt{\pi},0)$, $(\sqrt{\pi},\sqrt{\pi})$.

a. $-\pi$

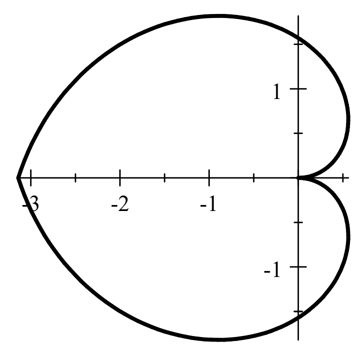
b. $-\sqrt{\pi}$

c. 1

d. $\sqrt{\pi}$

e. π

4. Find the area of the heart shaped region inside the polar curve $r = |\theta|$.



a. $\dfrac{\pi^3}{6}$

b. $\dfrac{\pi^3}{3}$

c. $\dfrac{4\pi^3}{3}$

d. $\dfrac{8\pi^3}{3}$

e. $\dfrac{16\pi^3}{3}$

5. The solid hemisphere $0 \le z \le \sqrt{4 - x^2 - y^2}$ has density $\delta = z$. Find the total mass.

a. $\pi/2$

b. π

c. 2π

d. 4π

e. 8π

6. The solid hemisphere $0 \le z \le \sqrt{4 - x^2 - y^2}$ has density $\delta = z$.

Find the z-component of the center of mass.

a. 1

b. $\frac{32}{15}\pi$

c. $\frac{64}{15}\pi$

d. $\frac{8}{15}$

e. $\frac{16}{15}$

7. Compute $\iiint \nabla \cdot \vec{F}\, dV$ on the solid cylinder bounded by

$x^2 + y^2 = 9$, $z = 0$ and $z = 5$ for the vector field $\vec{F} = (x^3, y^3, z(x^2 + y^2))$.

a. 45π

b. 90π

c. 360π

d. 810π

e. 900π

8. If $\vec{F} = (-yz, xz, z^2)$, compute $\vec{F} \cdot \vec{\nabla} \times \vec{F}$.

a. z^3

b. $z^3 - xyz$

c. $2z^3$

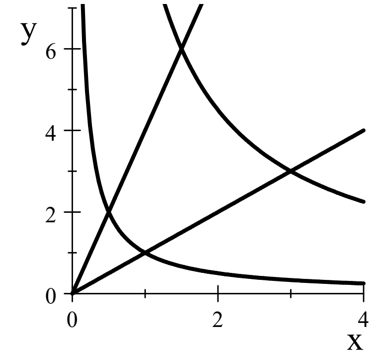
d. $2z^3 - 2xyz$

e. 0

9. Compute $\iint_R x^2\, dx\, dy$ over the diamond shaped region R bounded by

$$y = \frac{1}{x}, \qquad y = \frac{9}{x}, \qquad y = x, \qquad y = 4x$$

HINT: Use the curvilinear coordinates (u,v) where $x = uv$ and $y = \frac{u}{v}$.



a. -15

b. $-\frac{91}{36}$

c. $\frac{91}{6}$

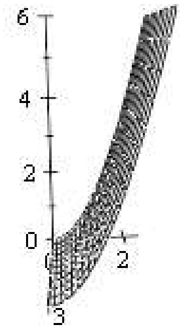
d. $\frac{91}{36}$

e. 15

Work Out: (Points indicated. Part credit possible. Show all work.)

10. $(20$ points$)$ Find the point $P = (x,y,z)$ on the plane $z = x + y - 3$ which is closest to the point $Q = (1,0,1)$. Find the distance from P to Q.

11. (24 points) Consider the sliding board given by $z = y^2$ for $0 \le x \le 3$, $y \ge 0$ and $z \le 6$.



- Find the mass of the slide if the surface mass density is $\delta = y$.
- If the slide is porous and rain is falling on the slide with velocity field $\vec{V} = (0, x, -y)$ find the flux of $\vec{V}$ **down** through the slide.

Parametrize the slide as $\vec{R}(u,v) = (u, v, v^2)$ and follow these steps:

a. Find the coordinate tangent vectors:

$\vec{e}_u =$

$\vec{e}_v =$

b. Find the normal vector and check its orientation:

$\vec{N} =$

After checking orientation:

$\vec{N} =$

c. Find the length of the normal vector:

$|\vec{N}| =$

d. Find the limits on u and v:

(continued)

e. Evaluate the density $\delta = y$ on the slide:

$$\delta\left(\vec{R}(u,v)\right) =$$

f. Compute the mass:

$$M =$$

g. Evaluate the velocity field $\vec{V} = (0, x, -y)$ on the slide:

$$\vec{V}\left(\vec{R}(u,v)\right) =$$

h. Compute the flux:

$$\iint \vec{V} \cdot d\vec{S} =$$