

Name _____

MATH 251 Exam 2B Fall 2016

Sections 504 P. Yasskin

1-9	/63
10	/20
11	/24
Total	/107

Multiple Choice: (7 points each. No part credit.)

1. The function $f = xy - \frac{2}{x} + \frac{4}{y}$ has a critical point at $(x,y) = (1,-2)$.

Use the Second Derivative Test to classify this critical point.

- a. Local Minimum
 - b. Local Maximum
 - c. Inflection Point
 - d. Saddle Point
 - e. Test Fails
2. Find the volume of the solid under $z = 2x^2y$ above the region in the xy -plane between $y = x$ and $y = x^2$.

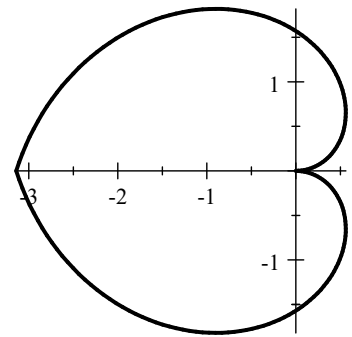
- a. $\frac{2}{35}$
- b. $\frac{35}{12}$
- c. $\frac{12}{35}$
- d. $\frac{1}{35}$
- e. $\frac{1}{12}$

3. Compute $\iint \sin(x^2) dx dy$ over the triangle with vertices $(0,0)$, $(\sqrt{\pi}, 0)$, $(\sqrt{\pi}, \sqrt{\pi})$.

- a. $-\pi$
- b. $-\sqrt{\pi}$
- c. 1
- d. $\sqrt{\pi}$
- e. π

4. Find the area of the heart shaped region inside the polar curve $r = |\theta|$.

- a. $\frac{\pi^3}{6}$
- b. $\frac{\pi^3}{3}$
- c. $\frac{4\pi^3}{3}$
- d. $\frac{8\pi^3}{3}$
- e. $\frac{16\pi^3}{3}$



5. The solid hemisphere $0 \leq z \leq \sqrt{4 - x^2 - y^2}$ has density $\delta = z$. Find the total mass.

- a. $\pi/2$
- b. π
- c. 2π
- d. 4π
- e. 8π

6. The solid hemisphere $0 \leq z \leq \sqrt{4 - x^2 - y^2}$ has density $\delta = z$.

Find the z -component of the center of mass.

- a. 1
- b. $\frac{32}{15}\pi$
- c. $\frac{64}{15}\pi$
- d. $\frac{8}{15}$
- e. $\frac{16}{15}$

7. Compute $\iiint \nabla \cdot \vec{F} dV$ on the solid cylinder bounded by

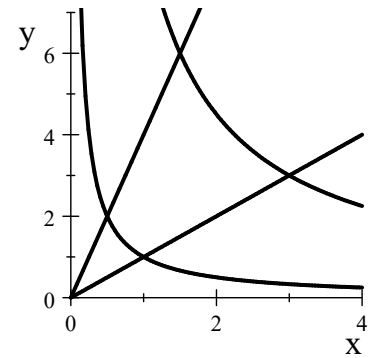
$x^2 + y^2 = 9$, $z = 0$ and $z = 5$ for the vector field $\vec{F} = (x^3, y^3, z(x^2 + y^2))$.

- a. 45π
- b. 90π
- c. 360π
- d. 810π
- e. 900π

8. If $\vec{F} = (-yz, xz, z^2)$, compute $\vec{F} \cdot \vec{\nabla} \times \vec{F}$.

- a. z^3
- b. $z^3 - xyz$
- c. $2z^3$
- d. $2z^3 - 2xyz$
- e. 0

9. Compute $\iint_R x^2 dx dy$ over the diamond shaped region R bounded by $y = \frac{1}{x}$, $y = \frac{9}{x}$, $y = x$, $y = 4x$
- HINT: Use the curvilinear coordinates (u, v) where $x = uv$ and $y = \frac{u}{v}$.



- a. -15
- b. $-\frac{91}{36}$
- c. $\frac{91}{6}$
- d. $\frac{91}{36}$
- e. 15

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (20 points) Find the point $P = (x, y, z)$ on the plane $z = x + y - 3$ which is closest to the point $Q = (1, 0, 1)$. Find the distance from P to Q .

11. (24 points) Consider the sliding board given by $z = y^2$ for $0 \leq x \leq 3$, $y \geq 0$ and $z \leq 6$.



- Find the mass of the slide if the surface mass density is $\delta = y$.
- If the slide is porous and rain is falling on the slide with velocity field $\vec{V} = (0, x, -y)$ find the flux of \vec{V} **down** through the slide.

Parametrize the slide as $\vec{R}(u, v) = (u, v, v^2)$ and follow these steps:

- a. Find the coordinate tangent vectors:

$$\vec{e}_u =$$

$$\vec{e}_v =$$

- b. Find the normal vector and check its orientation:

$$\vec{N} =$$

After checking orientation:

$$\vec{N} =$$

- c. Find the length of the normal vector:

$$|\vec{N}| =$$

- d. Find the limits on u and v :

(continued)

e. Evaluate the density $\delta = y$ on the slide:

$$\delta(\vec{R}(u,v)) =$$

f. Compute the mass:

$$M =$$

g. Evaluate the velocity field $\vec{V} = (0, x, -y)$ on the slide:

$$\vec{V}(\vec{R}(u,v)) =$$

h. Compute the flux:

$$\iint \vec{V} \cdot d\vec{S} =$$