Name____________________________

MATH 251 Final Exam Version A Fall 2016

Sections 504 P. Yasskin

1-13	/65	16	/10
14	/10	E.C.	/ 5
15	/20	Total	/110

Multiple Choice: (5 points each. No part credit.)

1. Find the point where the line $x = 2 + t, \quad y = 1 - t, \quad z = -2 + 2t$ intersects the plane $2x - y - z = 6$. At this point $x + y + z =$

a. 0
b. 1
c. 2
d. 3
e. 4

2. Find the plane tangent to the graph of $z = x^2 y$ at the point $(2,3)$. The z-intercept is

a. -24
b. -16
c. -4
d. 0
e. 4

3. Find the line perpendicular to the hyperboloid $z^2 - x^2 - y^2 = 3$ at the point $P = (2,3,4)$. This line intersects the xy-plane at

a. $(6,-8,0)$
b. $(-2,-3,0)$
c. $(8,6,0)$
d. $(4,6,0)$
e. $\left(\frac{8}{3},2,0\right)$

4. A box currently has length $L = 20$ cm which is increasing at 4 cm/sec, width $W = 15$ cm which is decreasing at 2 cm/sec, and height $H = 12$ cm which is increasing at 1 cm/sec. At what rate is the volume changing?

a. 3 cm^3/sec
b. 7 cm^3/sec
c. 540 cm^3/sec
d. 900 cm^3/sec
e. 3600 cm^3/sec

5. In order to hide from the Dark Invader, Duke Skywater flys the Millenium Eagle into a galactic dust storm. Currently, his position is $P = (10, -20, 30)$ and his velocity is $\vec{v} = (-4, 9, 2)$. He measures that currently the dust density is $\rho = 520$ and its gradient is $\vec{\nabla}\rho = (2, 1, -2)$. Find the current rate of change of the dust density as seen by Duke.

a. -60
b. 60
c. 517
d. 523
e. -3

6. Under the same conditions as in #5, in what **unit** vector direction should Duke travel to **increase** the dust density as quickly as possible?

a. $(2, 1, -2)$
b. $(-2, -1, 2)$
c. $\left(\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}\right)$
d. $\left(\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}\right)$
e. $\left(\frac{-2}{3}, \frac{-1}{3}, \frac{2}{3}\right)$

7. The point $(2,-1)$ is a critical point of the function $f = \dfrac{x^2y^2 + 2x - 4y}{xy}$.
Use the Second Derivative Test to classify the point.

a. Local Minimum
b. Local Maximum
c. Inflection Point
d. Saddle Point
e. Test Fails

8. Find the mass of the plate between $y = 2x$ and $y = x^2$ if the surface density is $\delta = xy$.

a. $\frac{8}{15}$
b. $\frac{16}{15}$
c. $\frac{32}{15}$
d. $\frac{8}{3}$
e. $\frac{16}{3}$

9. Find the y-component of the center of mass of the plate between $y = 2x$ and $y = x^2$ if the surface density is $\delta = xy$.

a. $\frac{8}{15}$
b. $\frac{5}{12}$
c. $\frac{12}{5}$
d. $\frac{15}{8}$
e. $\frac{32}{5}$

10. Compute $\oint \vec{F} \cdot d\vec{s}$ counterclockwise around the rectangle $0 \le x \le \pi$ and $0 \le y \le 2\pi$ for $\vec{F} = (y\sin x + \sin y, x\cos y + \cos x)$.

HINT: Use the Fundamental Theorem of Calculus for Curves or Green's Theorem.

a. -16π
b. -8π
c. 0
d. 8π
e. 16π

11. Compute $\int_{(0,0,0)}^{(1,2,0)} \vec{F} \cdot d\vec{s}$ for $\vec{F} = (2x + y, x + 2y, 2z)$ along the curve $\vec{r}(t) = (\sqrt{t}, t^2 + t^3, t^2 - t^3)$.

HINT: Find a scalar potential.

a. 1
b. 3
c. 5
d. 7
e. 9

12. Compute $\iint \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ over the sphere $x^2 + y^2 + z^2 = 4$ with outward normal for $\vec{F} = (x^2yz, xy^2z, xyz^2)$.

HINT: Use Stokes' Theorem or Gauss' Theorem.

a. 4π
b. 12π
c. $\frac{32}{3}\pi$
d. $\frac{64}{3}\pi$
e. 0

13. Compute $\iint \vec{F} \cdot d\vec{S}$ outward through the complete surface of the cylinder $x^2 + y^2 = 4$ for $1 \le z \le 4$ for $\vec{F} = (x^3, y^3, z(x^2 + y^2))$.

HINT: Use a Theorem.

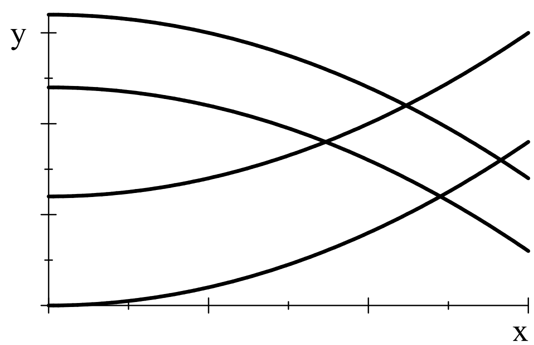
a. 96π
b. 72π
c. 48π
d. 24π
e. 18π

Work Out: (Points indicated. Part credit possible. Show all work.)

14. (10 points) Compute $\iint\limits_D x\,dx\,dy$ over the "diamond shaped" region D in the first quadrant bounded by the parabolas

$y = 16 - x^2$ $\quad$ $y = 12 - x^2$

$y = 6 + x^2$ and $y = x^2$



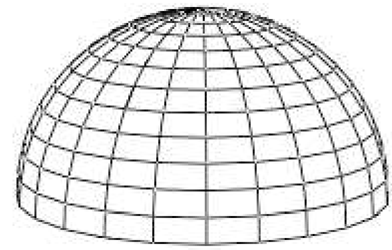
HINTS: Use the coordinates: $u = y + x^2$, $v = y - x^2$. Solve for x and y.

15. (20 points) Verify Stokes' Theorem $\iint_H \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial H} \vec{F} \cdot d\vec{s}$

for the vector field $\vec{F} = (-y, x, xz + yz)$

and the hemisphere $z = \sqrt{16 - x^2 - y^2}$ oriented up.

Be sure to check the orientation. Use the following steps:



a. RHS: Parametrize the boundary curve and compute the line integral:

$\vec{r}(\theta) =$

$\vec{v}(\theta) =$

$\vec{F}(\vec{r}(\theta)) =$

$\oint_{\partial H} \vec{F} \cdot d\vec{s} =$

(continued)

b. LHS: Compute the surface integral using the parametrization:

$$\vec{R}(\varphi,\theta) = (\quad 4\sin\varphi\cos\theta, \quad 4\sin\varphi\sin\theta, \quad 4\cos\varphi \quad)$$

$$\vec{\nabla} \times \vec{F} =$$

$$\left[\vec{\nabla} \times \vec{F}\right]_{\vec{R}(\varphi,\theta)} =$$

$$\vec{e}_\varphi =$$

$$\vec{e}_\theta =$$

$$\vec{N} =$$

$$\iint_H \vec{\nabla} \times \vec{F} \cdot d\vec{S} =$$

16. (10 points) Find 3 positive numbers x, y and z, whose sum is 90 such that $f(x,y,z) = xy^2z^3$ is a maximum.

Solve either by Eliminating a Variable or by Lagrange Multipliers.

5 points extra credit for doing both. Clearly separate solutions.

METHOD 1: Lagrange Multipliers:

METHOD 2: Eliminate a Variable: