

Vector Analysis Theorems

1. The **Fundamental Theorem of Calculus for Curves** states that if $\vec{r}(t)$ is a nice curve in $\mathbf{R}^n$ and f is a nice function in $\mathbf{R}^n$ then

$$\int_{\substack{A \\ \vec{r}}}^{B} \vec{\nabla} f \cdot d\vec{s} = f(B) - f(A)$$

2. **Green's Theorem** states that if R is a nice region in $\mathbf{R}^2$ and ∂R is its boundary curve traversed counterclockwise and P and Q are a nice functions on R then

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\, dy = \oint_{\partial R} P\, dx + Q\, dy$$

a. **2D Stokes' Theorem** states that if R is a nice region in $\mathbf{R}^2$ and ∂R is its boundary curve traversed counterclockwise and $\vec{F} = (P(x,y), Q(x,y), 0)$ is a nice vector field on R then

$$\iint_R \vec{\nabla} \times \vec{F} \cdot \hat{k}\, dx\, dy = \oint_{\partial R} \vec{F} \cdot d\vec{s}$$

b. **2D Gauss' Theorem** states that if R is a nice region in $\mathbf{R}^2$ and ∂R is its boundary curve traversed counterclockwise and $\vec{G} = (Q(x,y), -P(x,y), 0)$ is a nice vector field on R then

$$\iint_R \vec{\nabla} \cdot \vec{G}\, dx\, dy = \oint_{\partial R} \vec{G} \cdot d\vec{n}$$

3. **Stokes' Theorem** states that if S is a nice surface in $\mathbf{R}^3$ and ∂S is its boundary curve traversed counterclockwise as seen from the tip of the normal to S and $\vec{F}$ is a nice vector field on S then

$$\iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial S} \vec{F} \cdot d\vec{s}$$

4. **Gauss' Theorem** states that if V is a volume in $\mathbf{R}^3$ and ∂V is its boundary surface oriented outward from V and $\vec{F}$ is a nice vector field on V then

$$\iiint_V \vec{\nabla} \cdot \vec{F}\, dV = \iint_{\partial V} \vec{F} \cdot d\vec{S}$$